Course 22.314 “Structural Mechanics in Nuclear Power Technology”

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INTRODUCTORY DISCUSSION ON CREEP

1. Creep Stress-Strain Analysis

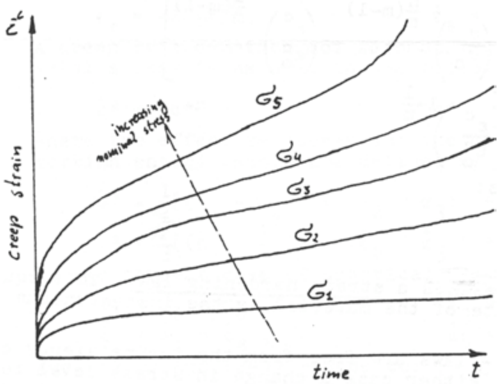

Fig. 1. Creep-strain - time curves for various stresses (Temperature T = const.)

If the creep curves in Fig.1 corresponding to various stress levels are assumed to be similar in shape, a general stress-strain-time relation can be written in the form

$$\varepsilon^c = f^*(\sigma)g^*(t)h^*(T), \qquad (1\text{-}1)$$

where $f^*(\sigma)$, $g^*(t)$ and $h^*(T)$, are arbitrary functions of stress, time, and temperature, respectively. The temperature dependence may also somehow be fitted into the parameters of the functions $f(\sigma)$ and $g(t)$, thus

$$\varepsilon^c = f(\sigma)g(t). \qquad (1\text{-}2)$$

The constant stress curves for metals are fit reasonably well in both the primary and secondary creep ranges power function forms for f and g, is, by

$$\frac{\varepsilon^c}{\varepsilon^c_o} = \left(\frac{\sigma}{\sigma_o}\right)^n \left(\frac{t}{t_o}\right)^m, \qquad (1\text{-}3)$$

where ε^c_o, σ_o and t_o are reference values of creep strain, stress and time chosen to produce dimensionless forms, and m and n are pure numbers. Any of these parameters could be temperature dependent.

Although Eq. (1-2) may fit constant stress curves adequately, there is inconsistency when it is applied to variable stress situations.
For example, assume that creep at stress σ_1 has progressed to ε^c_1 at t_1, then the stress is increased to $\sigma_2 > \sigma_1$, Eq. (1-3) predicts a sudden jump in creep strain. This is objectionable both theoretically and experimentally because creep strains, by definition, increase gradually with time.

This situation of the mathematical model can be obviated by expressing the creep constitutive equation in terms of strain rate $\dot{\varepsilon}^c = \dfrac{d\varepsilon^c}{dt}$, rather than strains.
Differentiating Eq. (1-3) we obtain:

$$\dot{\varepsilon}^c = \frac{m\varepsilon_o^c}{t_o}\left(\frac{\sigma}{\sigma_o}\right)^n\left(\frac{t}{t_o}\right)^{m-1} \qquad (1\text{-}4)$$

which can be expressed as

$$\dot{\varepsilon}^c = \beta\left(\frac{\sigma}{\sigma_o}\right)^n\left(\frac{t}{t_o}\right)^p \qquad (1\text{-}5)$$

When $p < 0$, we have a time-hardening law, if $p > 0$, Eq. (1-5) describes a material softening with time. This formulation may be useful if there are metallurgical changes in the material occurring as a function of time.
An alternate formulation is obtained by using Eq. (1-3) to eliminate time from Eq. (1-4).
According to Eq. (1-3),

$$\left(\frac{t}{t_o}\right)^m = \left(\frac{\sigma}{\sigma_o}\right)^{-n}\left(\frac{\varepsilon^c}{\varepsilon_o^c}\right), \qquad\qquad \left(\frac{t}{t_o}\right) = \left(\frac{\sigma}{\sigma_o}\right)^{-\frac{n}{m}}\left(\frac{\varepsilon^c}{\varepsilon_o^c}\right)^{\frac{1}{m}} \;;$$

substituting this in Eq. (1-4) gives:

$$\dot{\varepsilon}^c = \frac{m\varepsilon_o^c}{t_o}\left(\frac{\sigma}{\sigma_o}\right)^n\left[\left(\frac{\sigma}{\sigma_o}\right)^{-\frac{n}{m}(m-1)}\left(\frac{\varepsilon^c}{\varepsilon_o^c}\right)^{\frac{1}{m}(m-1)}\right]$$

$$\dot{\varepsilon}^c = \frac{m\varepsilon_o^c}{t_o}\left(\frac{\sigma}{\sigma_o}\right)^{\frac{n}{m}}\left(\frac{\varepsilon^c}{\varepsilon_o^c}\right)^{1-\frac{1}{m}} \qquad (1\text{-}6)$$

which can be expressed as:

$$\dot{\varepsilon}^c = C\left(\frac{\sigma}{\sigma_o}\right)^q\left(\frac{\varepsilon^c}{\varepsilon_o^c}\right)^r \qquad (1\text{-}7)$$

If $r < 0$, Eq. (1-7) represents a strain hardening law. This equation uniquely defines the state of the material by the instantaneous values of σ and ε^c.
These creep strain rate-laws are free from the inconsistency of the creep strain-law. In either case a change in stress level results in an instantaneous change in creep strain rate, but not in an instantaneous change in creep strain.
However, one deficiency still exists. Both equations predict zero creep strain if stress is suddenly reduced to zero, whereas the phenomenon of creep recovery is known from experiments.
These creep laws neglect recovery effects.

2. Creep of an Internally Pressurized Thick-Walled Sphere

The equilibrium equation in spherical symmetry is:

$$\frac{d\sigma_r}{dr} + \frac{2}{r}\left(\sigma_r - \sigma_\theta\right) = 0. \qquad (2\text{-}1)$$

The relations between rate of strain and rate of radial displacement in spherical symmetry are:

$$\dot{\varepsilon}_r = \frac{d\dot{u}}{dr}, \qquad \dot{\varepsilon}_\theta = \frac{\dot{u}}{r}. \tag{2-2a,b}$$

The assumption of incompressibility in the inelastic range, i.e. no volume change in creep (or plastic deformation),

$$\dot{\varepsilon}_r + \dot{\varepsilon}_\theta + \dot{\varepsilon}_\theta = 0 \tag{2-3}$$

gives a relation between the radial and tangential strain rates:

$$\dot{\varepsilon}_r = -2\dot{\varepsilon}_\theta. \tag{2-3a}$$

Substituting Eqs. (2-3a) and (2-2b) into Eq. (2-2a) gives a differential equation for the radial displacement rate:

$$\frac{d\dot{u}}{dr} + \frac{2}{r}\dot{u} = 0 \tag{2-4}$$

Integration yields

$$\dot{u} = C_1 r^{-2}, \qquad C_1=\text{const.}, \tag{2-5}$$

And substituting into Eqs. (2-2a,b) gives:

$$\dot{\varepsilon}_r = -2C_1 r^{-3}, \qquad \dot{\varepsilon}_\theta = C_1 r^{-3}. \tag{2-6}$$

A creep rate equation for secondary creep at a given temperature in a multi-axial system is assumed in the following form:

$$\dot{\varepsilon}^* = \lambda \sigma^{*n} \qquad \text{or} \qquad \sigma^* = \left(\frac{\dot{\varepsilon}^*}{\lambda}\right)^{\frac{1}{n}}, \tag{2-7}$$

where the effective stress and the effective strain rate based on the distortion energy are defined by

$$\sigma^* = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{\frac{1}{2}}, \tag{2-8}$$

$$\dot{\varepsilon}^* = \frac{\sqrt{2}}{3}\left[(\dot{\varepsilon}_1 - \dot{\varepsilon}_2)^2 + (\dot{\varepsilon}_2 - \dot{\varepsilon}_3)^2 + (\dot{\varepsilon}_3 - \dot{\varepsilon}_1)^2\right]^{\frac{1}{2}}. \tag{2-9}$$

In the present case of spherical symmetry, i.e.

$$\sigma_1 = \sigma_2 = \sigma_\theta, \qquad \sigma_3 = \sigma_r,$$

$$\dot{\varepsilon}_1 = \dot{\varepsilon}_2 = \dot{\varepsilon}_\theta, \qquad \dot{\varepsilon}_3 = \dot{\varepsilon}_r,$$

Eqs. (2-8) and (2-9) reduce to:

$$\sigma^* = \sigma_\theta - \sigma_r \tag{2-8a}$$

$$\dot{\varepsilon}^* = \frac{2}{3}(\dot{\varepsilon}_\theta - \dot{\varepsilon}_r). \tag{2-9a}$$

Combining Eqs. (2-3a) and (2-9a) gives

$$\dot{\varepsilon}^* = 2\dot{\varepsilon}_\theta, \tag{2-10}$$

And combining Eqs. (2-10) and (2-2b) with Eq. (2-5) gives

$$\dot{\varepsilon}^* = 2C_1 r^{-3}. \tag{2-11}$$

Substituting Eq. (2-8a) into the equilibrium equation, Eq. (2-1),

$$\frac{d\sigma_r}{dr} = \frac{2}{r}\sigma^*$$

and utilizing the creep law, Eq. (2-7), together with Eq. (2-11) yields

$$\frac{d\sigma_r}{dr} = 2\left(\frac{2C_1}{B}\right)^{\frac{1}{n}}(r)^{-\frac{3}{n}-1} \tag{2-12}$$

and by integration

$$\sigma_r = -2\frac{n}{3}\left(\frac{2C_1}{B}\right)^{\frac{1}{n}}(r)^{-\frac{3}{n}} + C_2. \tag{2-13}$$

Designating

$$C = -2\frac{n}{3}\left(\frac{2C_1}{B}\right)^{\frac{1}{n}}, \tag{2-14}$$

Eq. (2-13) becomes

$$\sigma_r = Cr^{-\frac{3}{n}} + C_2. \tag{2-13a}$$

Utilizing Eq. (2-14) in Eq. (2-12) gives

$$\frac{d\sigma_r}{dr} = -\frac{3}{n}C(r)^{-\frac{3}{n}-1}. \tag{2-12a}$$

Substituting Eqs. (2-12a) and (2-13a) into the equilibrium equation, Eq. (2-1), gives a relation for the tangential stress:

$$\sigma_\theta = C\left(1 - \frac{3}{2n}\right)r^{-\frac{3}{n}} + C_1 \tag{2-15}$$

With internal and external pressures acting on the inside and outside faces of the hollow sphere, defined by $r = r_i$ resp. $r = r_o$, the boundary conditions are:

$$\sigma_r\big|_{r=r_i} = -p_i, \qquad \sigma_r\big|_{r=r_o} = -p_o. \tag{2-16}$$

Applying these boundary conditions to Eq. (2-13) gives the following expression for the radial creep stress:

$$\sigma_r = (p_i - p_o)\frac{\left(\frac{r_o}{r_i}\right)^{\frac{3}{n}} - \left(\frac{r_o}{r}\right)^{\frac{3}{n}}}{\left(\frac{r_o}{r_i}\right)^{\frac{3}{n}} - 1}. \tag{2-17}$$

Differentiating Eq. (2-17),

$$\frac{d\sigma_r}{dr} = \frac{3(p_i - p_o)}{nr}\left(\frac{r_o}{r}\right)^{\frac{3}{n}}\left[\left(\frac{r_o}{r_i}\right)^{\frac{3}{n}} - 1\right]^{-1}. \tag{2-18}$$

Combining the creep rate equation, Eq. (2-7), the relations for the effective strain rate, Eqs. (2-9a) and (2-10), and Eq. (2-18), gives the following expression for the tangential creep strain rate:

$$\dot{\varepsilon}_\theta = \frac{B}{2}\left[\frac{\frac{3}{2n}(p_i - p_o)\left(\frac{r_o}{r}\right)^{\frac{3}{n}}}{\left(\frac{r_o}{r_i}\right)^{\frac{3}{n}} - 1}\right]^n, \tag{2-19}$$

The tangential creep strain rate at the inner surface, $r = r_i$, is

$$\dot{\varepsilon}_{\theta}\Big|_{r=r_i} = \frac{dr_i}{r_i\,dt} = \frac{\dot{\varepsilon}_0}{2}\left(\frac{\frac{3}{2n}\left(p_i - p_o\right)}{1-\left(\frac{r_i}{r_o}\right)^{\frac{3}{n}}}\right)^{n}. \qquad (2\text{-}20)$$

3. <u>Bibliography</u>

See e.g. :
F.K.G. Odquist, J. Hult: Kriechfestigheit metallischer Werkstoffe. Springer-Verlag, Berlin (1962).