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TWO-BAR MECHANISM MATHEMATICAL MODEL FOR LINEAR-CREEP RATCHETTING OF THIN-WALLED CYLINDRICAL FUEL ELEMENT CLADDINGS SUBJECTED TO INTERNAL PRESSURE AND THERMAL CYCLINC

Problem: Creep ratchetting due to cyclic thermal stresses
Geometry: Thin-walled cylindrical shell represented by two-bar mechanism
Material: Linear viscous
Method: Analytical

1. Introduction

In a nuclear reactor environment, a fuel element cladding is subject to both pressure loadings and cyclic thermal stresses. The pressure force on the fuel element are caused by the external coolant flow acting on the outside surface of the cladding and the pressure forces due to fuel swell and fission gas release acting on the inside. The thermal stresses are caused by the temperature gradient that exists across the cladding wall and is related to the neutron flux; hence, it varies cyclically with the power output requirements of the reactor.

When a fuel element is thermally cycled under a sustained pressure it can experience cumulating inelastic deformations, a process which is called ratchetting. This phenomenon occurs when uni-directional inelastic strains are produced during each temperature cycle and these, because of their irreversible nature, have a cumulative effect which causes progress growth of the clad in both the radial and axial directions. Ratchetting can restrict coolant flow and can ultimately lead to failure of fuel ?elements?.

The first analytical treatment concerning the ratchetting phenomenon by D.R. MILLER investigated the ratchet mechanism associated with ?combining? thermal and mechanical stresses in thin-walled cylindrical pressure vessel for cases of heat flow with and without heat generation within the walls By using a simple uniaxial model which represents the behavior of the ? stresses in the cylinder wall, MILLER derives a criterion for the incremental growth when the material ratchets due to plastic deformations.

J. BREE presented an analysis in which the inelastic strains developed by thermal cycling were caused by both yielding and creep. BREE’s theory, however, does not include the effects of material property dependence on temperature and irradiation, and requires that a steady-state stress cycle established across the clad thickness before an estimation can be made with regards

to the incremental growth per cycle. Furthermore, the ? requires that the pressure forces in the clad remain constant throughout the cycling process.
HIBBELER and MURA developed an analysis of the creep ratchetting problem which includes those effects which BREE has neglected to take into count. This analysis assumes linear viscoelastic behavior of the cladding material.

2. Development of a Two-Bar Mechanism Model

Provided the cylindrical cladding of a sealed reactor fuel rod is considered thin, and the assumption of plane strain holds, the distribution of circumferential stress σ_θ and axial stress σ_z across the thickness of the clad wall when subjected to a net internal pressure, $p = p_o - p_i$ becomes uniform having the magnitude

$$\sigma_\theta = \frac{pR}{t}, \qquad \sigma_Z = \frac{pR}{2t} \tag{1a,b}$$

The effect of σ_r is negligible. In these equations t is the thickness of the clad, and R is the radius measured to the mean thickness of the clad.

For a thin cylindrical shell having a temperature drop ΔT across its wall, the thermal stress distribution is linear, having the extreme values

$$\sigma_\theta = \sigma_Z = \pm \frac{E\alpha\Delta T}{2(1-v)} \tag{2}$$

at the inner and outer surfaces of the cladding. E, v, and α are the module of elasticity, the Poisson's ratio, and coefficient of thermal expansion, respectively, of the clad material.

Since even with such a simple biaxial stress distribution an analysis ? the creep ratchetting phenomenon is complicated, R. HIBBELER and T. MURA in their study of creep ratchetting of cylindrical fuel element claddings further simplified the analysis by considering the hoop stresses as the ? significant stresses in causing creep ratchetting. For a first approximation this assumption appears justifiable since relatively large fuel swelling and gas pressures develop in comparison with the imposed thermal stress distribution. The influence of neglecting the effect of σ_z in comparison with σ_θ in the following analysis will lead to pessimistic results, since the effect of the hoop stress opposes the axial stress in deforming the cladding wall. A positive hoop stress acting alone on the clad will cause it to expand in the radial direction, whereas a positive axial stress causes radial contraction of the clad.

Consider a section of the cylindrical cladding as shown in Fig. 1a. The considerably simplifying mathematical-mechanical two-bar model as proposed by R. HIBBELER and T. MURA and shown in Fig. 1c assumes uniform stra? behavior of the cylindrical cladding by requiring the two bars A and B, having equal cross-sectional areas, to be jointly attached to a fixed wall and weightless rigid bar guided by a roller mechanism.

In Fig. lb, the stresses due to uniform pressure load and thermal gradient are shown. The linear thermal stress distribution has been replaced by an average uniform stress distribution of magnitude $\sigma_t = \frac{E\alpha\Delta T}{4(1-v)}$.

In this way the same magnitude of force F is preserved. Besides inducing thermal stresses, the temperature gradient acting through the cladding thickness filters the mechanical properties of the cladding material. For this reason the behavior of the cladding material at the outer region B of the clad, see Fig. 1a, will be different from the material behavior at the inner reg A. The correspondence of the material behavior is preserved in the mode by requiring bars A and B to have material properties corresponding to the averaged materials properties in the regions A and B, respectively, of the cladding.

Correspondence between the behavior of the cylindrical cladding and ? two-bar model requires that the initial stress distribution in the clad equation that of the model just after the initial temperature gradient is applied. With reference to Fig. 1b, this requires that we satisfy the equations:

(initial cladding stress) = (initial model stress)

$$\frac{p_C R}{t} + \frac{E_C\alpha\Delta T}{4(1-v)} = \frac{W}{2A} + \frac{(\alpha T)_A - (\alpha T)_B}{2}E$$

$$\frac{p_C R}{t} + \frac{E_C\alpha\Delta T}{4(1-v)} = \frac{W}{2A} - \frac{(\alpha T)_A - (\alpha T)_B}{2}E \tag{3}$$

where

A = cross-sectional area of each bar of the two-bar mechanism,
W = loading on the two-bar model,
subscripts A, B = reference to the members A, resp. B,
subscript c = reference to cladding.

The behavior of the cladding can in this way be represented by the two-bar model which is analyzed in the following to determine the ratchet growth of the cladding in the circumferential direction due to thermal cycling.

3. Analysis of the Two-Bar Mechanism Using a Linear Steady-State Creep ?

The creep law to be considered in this section is of the form

$$\dot{\varepsilon} = K(\sigma - \sigma_o), \tag{4}$$

where K and σ_o are assumed known constants. If $\sigma_o = 0$, this equation would describe the secondary creep behavior of a material whose strain rate variation with stress is linear. The constant σ_o is included in the above creep law in order that the results of the linear analysis can be used in place of an arbitrary creep law analysis, a method for which will be described later.

The imposed temperature distribut~on in the members is shown in Fig?.

First Part of the 1st cycle

The following relations must be satisfied for the mechanism during the time period $0 < t < t_1$:

Equilibrium equation: $\sigma_A + \sigma_B = \frac{W}{A}$ (5)

Compatibility equation: $\varepsilon_A = \varepsilon_B$ (6)

Total Strain for each member: $\varepsilon = \frac{\sigma}{E} + \alpha T + \varepsilon^c$ (7)

Steady-State Creep Law: $\dot{\varepsilon}^c = K(\sigma - \sigma_O)$ (8)

Taking the derivative of Eq. (7) and noting that (αT) and E are constants during the time period yields:

$$\dot{\varepsilon} = \frac{1}{E}\frac{d\sigma}{dt} + K(\sigma - \sigma_O). \tag{9}$$

Combining the equilibrium and compatibility equations, Eqs. (5,6), for the stress in member B, with Eq. (9) gives the following stress-time different equation

$$\frac{d\sigma_B}{dt} + C_2 E \sigma_B = (C_3 + C_4) K_A E \tag{10}$$

where

$$C_2 = \frac{K_A + K_B}{2},\ C_3 = \frac{W}{2A},\ C_4 = \frac{1}{2}\frac{K_B \sigma_{O_B} - K_A \sigma_{O_A}}{K_A} \tag{10a}$$

Solving this equation, using the initial condition $\sigma_B = \frac{1}{C_{B_1}}$, at t

gives $$\sigma_B = \frac{C_3 + C_4}{C_2} K_A + \left[\frac{1}{\sigma_B} - \frac{C_3 + C_4}{C_2} K_A\right] e^{-C_2 E t}, \tag{11}$$

where the superscript 1 refers to the first part of the temperature cycle. To determine the initial value $\frac{1}{\sigma_{B_1}}$ use the strain relation at time t = 0. Noting that $\varepsilon^c = 0$ we obtain

$$\varepsilon_A = \frac{\sigma^1_{A_1}}{E} + (\alpha T)_A,\ \varepsilon_B = \frac{\sigma^1_{B_1}}{E} + (\alpha T)_B,$$

From the equilibrium and compatibility relations, Eqs. (5,6), we get

$$\sigma^{1}_{B_1} = C_1 E + C_3$$

where

$$C_1 = \frac{(\alpha T)_A - (\alpha T)_B}{2} \qquad (12)$$

and C_3 is defined by Eq. (10a). Substituting Eq. (9) into Eq. (7), integrating, using the initial condition $\varepsilon^{C}_{B} = 0$ at time t = 0 and simplifying yields

$$\varepsilon^{C}_{B} = \frac{C_3 - C_5}{C_2} K_B K_A t + \frac{K_B}{C_2 E}\left(\sigma^{1}_{B_1} - \frac{C_3 + C_4}{C_2} K_A\right)\left(1 - e^{-C_2 E t}\right) \qquad (13)$$

where

$$C_5 = \frac{\sigma_{O_A} - \sigma_{O_B}}{2}.$$

In a similar manner it can be shown that

$$\varepsilon^{C}_{A} = 2K_A C_3 t - \frac{K_A t}{C_2}\left(C_3 K_A + K_B C_3\right) - \frac{K_B}{C_2 E}\left(\sigma^{1}_{B_1} - \frac{C_3 + C_4}{C_2} K_A\right)\left(1 - e^{-C_2 E t}\right) \qquad (14)$$

Substituting Eqs. (11, 13) into Eq. (7), and using Eq. (6) gives the total strain-time relation for the two-bar mechanism as

$$\varepsilon = \frac{C_3 + C_4}{C_2}\frac{K_A}{E} + \frac{1}{E}\left(\sigma^{1}_{B_1} - \frac{C_3 + C_4}{C_2} K_A\right)\left(1 - \frac{K_B}{C_2}\right) e^{-C_2 E t} + (\alpha T)_B$$
$$+ \frac{C_3 - C_5}{C_2} K_A K_B t + \frac{K_B}{C_2 E}\left(\sigma^{1}_{B_1} - \frac{C_3 + C_4}{C_2} K_A\right). \qquad (15)$$

These established equations are valid only during the time period $0 < t < t_1$. When $t = t_1$, the temperature in the members is suddenly change so that a jump in the stress distribution will occur.

Second Part of the 1st Cycle

During the second part of the 1st cycle the expression for total stra must include the previous inelastic creep strains that have developed in mechanism during the first part of the 1st cycle. Thus Eq. (7) becomes

$$\varepsilon = \frac{\sigma}{E'} + (\alpha T)' + K(\sigma - \sigma_o) + \sum \varepsilon^{C}. \qquad (16)$$

The primed constants designate the material properties for the lower temperature part of the cycle, and the term $\Delta\varepsilon^c$ herein represents the in-elastic creep strain accumulated during the first part of the 1st cycle. These values of $\Delta\varepsilon^c$ are given from Eqs. (13, l4) for the time $t = t_1$.

The analysis used to describe the behavior of the mechanism during the second part of the 1st cycle, and for all further cycles, follows the procedure used for the first part of the 1st cycle. The only difference is that all the inelastic creep strains that have occurred during the previous time periods have to be included when using the total strain equation.

Assuming a constant loading on the mechanism the relations which describe the behavior of the mechanism during the n^{th} cycle are given below.

First Part of the n^{th} Cycle

Stress

$$\sigma_B = \frac{C_3 + C_4}{C_2} K_A + \left[\sigma^1_{B_n} - \frac{C_3 + C_4}{C_2} K_A\right] e^{-C_2 E t} \tag{17}$$

Strain

$$\begin{aligned}
\varepsilon = {} & \frac{C_3 + C_4}{C_2}\frac{K_A}{E} + \frac{1}{E}\left[\sigma^1_{B_n} - \frac{C_3 + C_4}{C_2} K_A\right]\left[1 - \frac{K_B}{C_2}\right] e^{-C_2 t} + (\alpha T)_B + \frac{C_3 - C_5}{C_2} K_A K_B t \\
& + \frac{K_B}{C_2 E}\left[\sigma^1_{B_n} - \frac{C_3 + C_4}{C_2} K_A\right] + \frac{C_3 - C_5}{C_2}(n-1) K_A K_B t_1 + \frac{C_3' - C_5'}{C_2'}(n-1) K_A' K_B' t_2 \\
& + \frac{K_B}{C_2 E}\left(1 - e^{-C_2 t}\right)\sum_{i=1}^{n-1}\left[\sigma^1_{B_n} - \frac{C_3 + C_4}{C_2} K_A\right] + \frac{K_B'}{C_2' E'}\left(1 - e^{-C_2' E' t_2}\right)\sum_{i=1}^{n-1} \\
& \left[\sigma^1_{B_n} - \frac{C_3' + C_4'}{C_2'} K_A'\right].
\end{aligned} \tag{18}$$

Second Part of the n^{th} Cycle

Stress

$$\sigma_B = \frac{C_3' - C_4'}{C_2'} K_A' + \left[\sigma^1_{B_N} - \frac{C_3 + C_4'}{C_2'} K_A'\right] e^{-C_2' E' t} \tag{19}$$

Strain

$$
\begin{aligned}
\varepsilon = {} & \frac{C_3 + C_4 E'}{C_2'}\frac{K_A'}{E'} + \frac{1}{E'}\left[{}^{2}\sigma_{B_n} - \frac{C_3 + C_4 E'}{C_2'}K_A'\right]\left[1 - \frac{K_B'}{C_2'}\right]e^{-C_2' E' t} + (\Delta T)\beta_B \\
& + \frac{C_3 - C_5'}{C_2'}K_A' K_B' t + \frac{K_B'}{C_2' E'}\left[{}^{2}\sigma_{B_n} - \frac{C_3 + C_4 E'}{C_2'}K_A'\right] + \frac{C_3 - C_5}{C_2} n K_A K_B t_1 \\
& + \frac{C_3 - C_5'}{C_2'}(n-1)K_A' K_B' t_2 + \frac{K_B}{C_2 E}\left(1 - e^{-C_2 E t_1}\right)\sum_{i=1}^{n}\left[{}^{1}\sigma_{B_i} - \frac{C_3 + C_4}{C_2}K_A\right] \\
& + \frac{K_B'}{C_2' E'}\left(1 - e^{-C_2' E' t_2}\right)\sum_{i=1}^{n-1}\left[{}^{2}\sigma_{B_i} - \frac{C_3 + C_4 E'}{C_2'}K_A'\right].
\end{aligned} \tag{20}
$$

The initial stress values can be found from the following relations

$$
\begin{aligned}
{}^{1}\sigma_{B_n} = {} & \frac{C_3 + C_4 E'}{C_2'}\frac{E}{E'}K_A'\frac{\left(1 - e^{-C_2' E' t_2}\right)\left(1 - e^{-(n-1)\Phi}\right)}{\left(1 - e^{-\Phi}\right)} \\
& + \frac{C_3 + C_4}{C_2}K_A\frac{\left(1 - e^{-C_2 E t_1}\right)\left(e^{-C_2' E' t_2}\right)\left(1 - e^{-(n-1)\Phi}\right)}{\left(1 - e^{-\Phi}\right)} \\
& + \left(C_3 + C_1 E\right)\frac{\left(1 - e^{-n\Phi}\right) - e^{-C_2' E' t_2}\left(1 - e^{-(n-1)\Phi}\right)}{\left(1 - e^{-\Phi}\right)},
\end{aligned} \tag{21a}
$$

$$
\begin{aligned}
{}^{2}\sigma_{B_n} = {} & \frac{C_3 + C_4 E'}{C_2'}K_A'\frac{\left(1 - e^{-C_2' E' t_2}\right)\left(e^{-C_2 E t_1}\right)\left(1 - e^{-(n-1)\Phi}\right)}{\left(1 - e^{-\Phi}\right)} \\
& + \frac{C_3 + C_4}{C_2}\frac{E'}{E}K_A\frac{\left(1 - e^{-C_2 E t_1}\right)\left(1 - e^{-n\Phi}\right)}{\left(1 - e^{-\Phi}\right)} + \left(C_3 + C_1' E'\right)\frac{\left(1 - e^{-n\Phi}\right) - e^{-C_2 E t_1}\left(1 - e^{-(n-1)\Phi}\right)}{\left(1 - e^{-\Phi}\right)} \\
& - \left(\frac{C_3 E'}{E} + C_1 E'\right)\frac{\left(1 - e^{-C_2 E t_1}\right)\left(1 - e^{-n\Phi}\right)}{\left(1 - e^{-\Phi}\right)},
\end{aligned} \tag{21b}
$$

where $\Phi = \left(C_2 E t_1 + C_2' E' t_2\right)$.

From Eq. (20), with $t = t_1$ we can def~ne the strain in the mechanism at the end of the nth cycle of temperature. Substituting in the initial stress values, Eq. (21), and simplifying, we obtain a closed form solution for the total accumulated strain ε in the model at the end of the nth cycle.

$$\varepsilon_n = \frac{n(C_3 - C_5)}{C_2} K_A K_B t_1 + \frac{n(C_3' - C_5')}{C_2'} K_A' K_B' t_1 + \frac{(\alpha T)_A + (\alpha T)_B}{2} + \frac{C_3}{E'}$$

$$+\left[\left(\frac{C_3'}{E'}\left(\frac{K_A'}{C_2'}-1\right)+\frac{K_A C_4'}{C_2' E'}-C_1'\right)-\left(\frac{C_3}{E}\left(\frac{K_A}{C_2}-1\right)+\frac{K_A C_4}{C_2 E}-C_1\right)\right]\left(1-e^{-C_2' E' t_1}\right)\left(\frac{K_B}{C_2}\right)$$

$$-\left(\frac{K_B'}{C_2'}\right)\frac{(n-1)-ne^{-\lambda t_1}+e^{-n\lambda t_1}}{\left(1-e^{-\lambda t_1}\right)^2}+\left(1-\frac{K_B}{C_2}\right)\frac{1-e^{-n\lambda t_1}}{1-e^{-\lambda t_1}}+\left(\frac{C_3}{E}\left(\frac{K_A}{C_2}-1\right)+\frac{K_A C_4}{C_2 E}-C_1\right] \qquad (22)$$

$$X\left[\left(1-e^{-C_2 E t_1}\right)\left(1-\frac{K_B}{C_2}\right)+e^{-C_2 E t_1}\left(1-e^{-C_2' E' t_1}\right)\left(1-\frac{K_B'}{C_2'}\right)\right]\frac{1-e^{-n\lambda t_1}}{1-e^{-\lambda t_1}},$$

where $\lambda = \left(C_2 E + C_2' E'\right)$.

4. Influence of the Material Parameters on the Ratchetting Process

Having developed the time-dependent stress and strain relations describing the ratchetting behavior in the two-bar mechanism for the nth cycle we are now in a position to discuss the influence of the loading and mate constants.

For simplification, if we consider the above equations without a loading, i.e. p = 0; and with $\sigma_O = \sigma_O' = (\alpha T)_A' = (\alpha T)_B' = 0$, and $t_1 = t_2$, Eqs. (17) – (21) reduce to a simplified form. The stress behavior in membe then plots as that shown in Fig. 3. After infinite temperature cycli the stress cycle reaches a steady-state cycle, whose initial stress values can be calculated from Eq. (21) for $n \to \infty$.

The magnitude of the jump in stress during temperature changes is dependent on the relative difference in the thermal strains in the material and is influenced by the difference in the elastic modulus, $(E - E')$ and the initial stress values. For $E \approx E'$ the magnitude of the jump J is simplified and becomes a constant value

$$J = \frac{(\alpha T)_A - (\alpha T)_B}{2} E'. \qquad (23)$$

From Eq. (22), we can find the $\lim_{n\to\infty} \varepsilon_n$. It is found that:

$\varepsilon \to +\infty$ if $\frac{K_A'}{K_B'} > \frac{K_A}{K_B}$ (positive ratchetting)

$\varepsilon \to -\infty$ if $\frac{K_A}{K_B} > \frac{K_A'}{K_B'}$ (negative ratchetting)

$\varepsilon = 0$ if $K_A = K_B, K_A' = K_B'$. (no growth)

Recalling the form of the linear creep law used here, i.e. $\dot{\epsilon} = K\sigma$, and the fact that $K_A \neq K_A'$, $K_B \neq K_B'$. The inequality condition for positive ratchetting for example, has the following interpretation: When the temperature gradient is imposed, causing compressive stresses in A and tensi stresses in B, we require that the relative creep rate of bar A to bar B be less than the relative creep rate when bar A is in tension and bar B is in compression.

The strain behavior of the system ratchetting under thermal cycling and constant internal pressure p_O given by Eqs. (l8) and (20) is illustrated in Fig. 4. But when a fuel element is thermally cycled, it has to be assumed that fuel swelling and fission gas release occurs during the interval at which the temperature is changing. Thus any increment of inter pressure loading on the mechanism would occur during the instant of rise fall of the temperature gradient. The effect of this is to increase the slopes of the total strain curves which describe the behavior for each half-cycle. Thus the effect of load increase during cycling increases the incremental growth per cycle.

If one considers a more general, non-linear creep law in the analysis rather than the linear creep analysis presented here, the observations made in regard to the stress and stress-jump behavior would be the same. The main difference lies in the fact that a non-linear analysis accounts for greater stress relaxation during the temperatures cycles.

5. Bibliography

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Figure 1

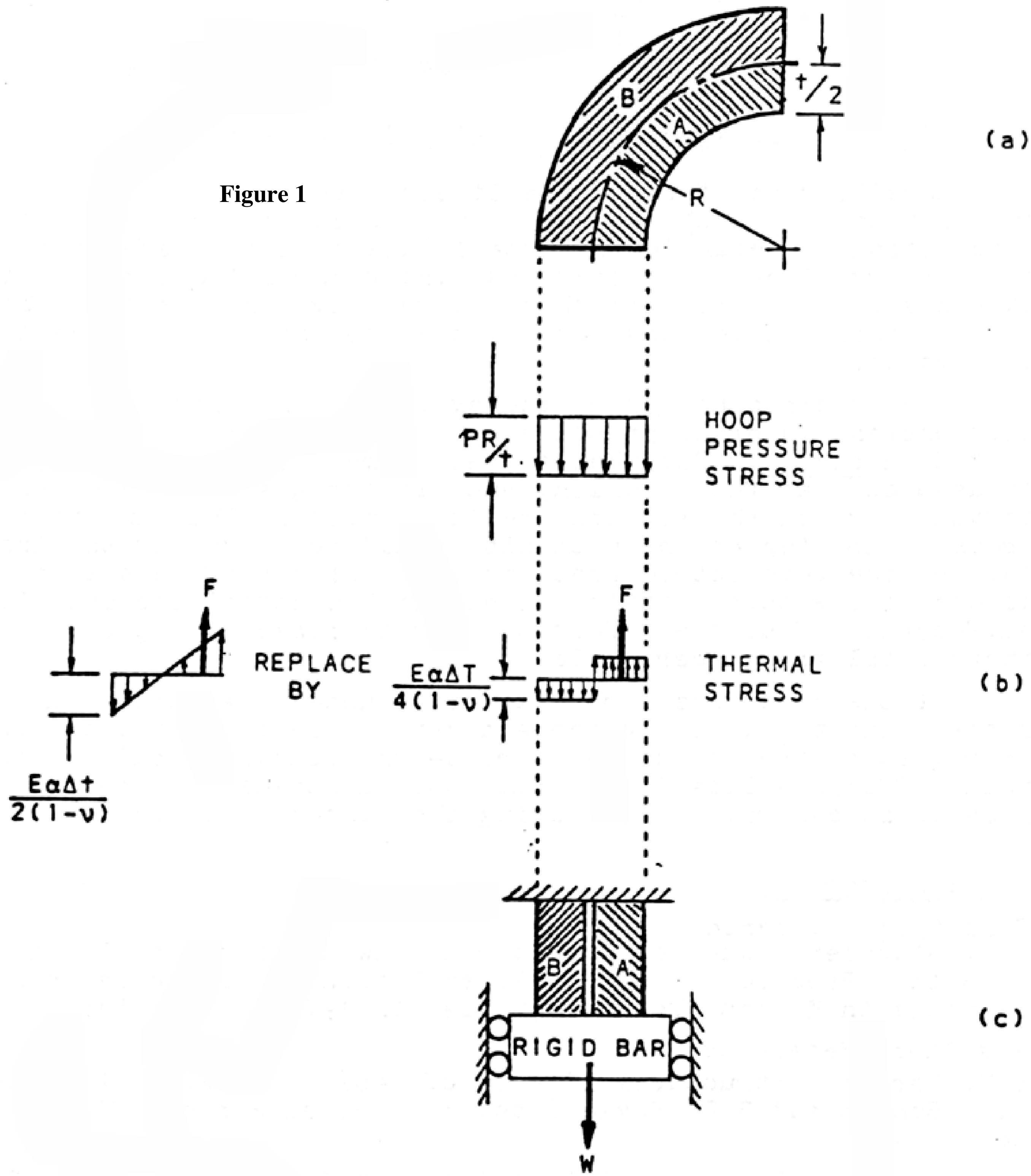
t/2
B
A
R
(a)
PR/t
HOOP PRESSURE STRESS
F
REPLACE BY
EαΔT / 4(1-ν)
THERMAL STRESS
(b)
EαΔt / 2(1-ν)
B
A
RIGID BAR
W
(c)

Figure 2

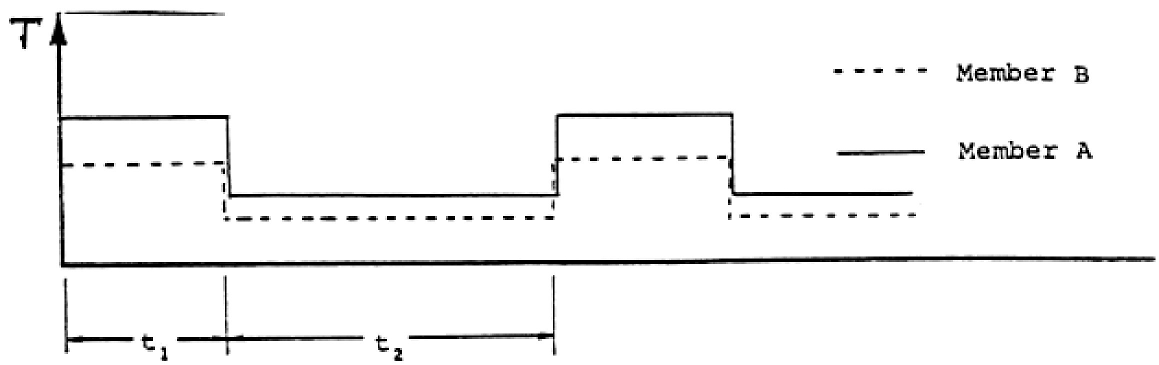

Figure 3

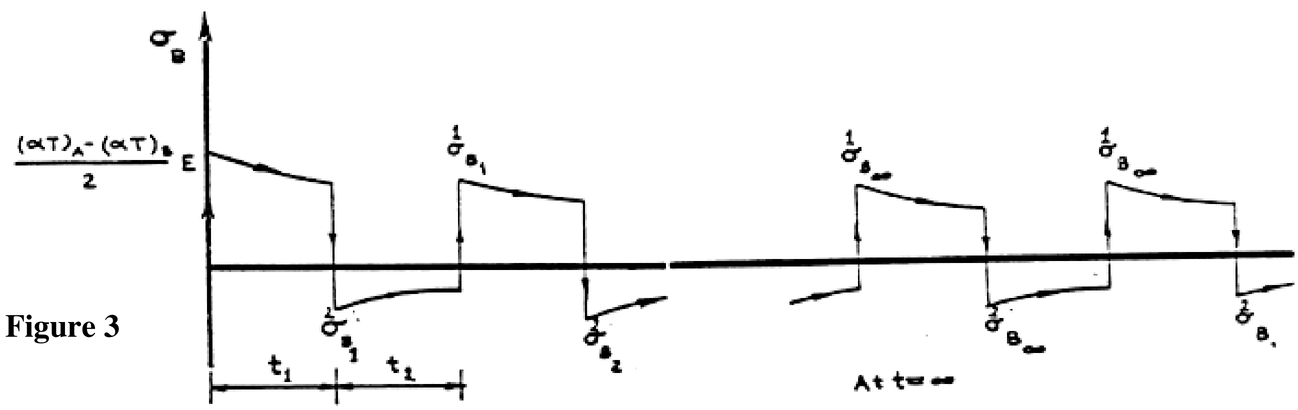

Figure 4