NOTES ON BEAM THEORY
J.E. Meyer
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<u>Beam Geometry and Notation</u>: Consider a beam with the beam axis in the z-direction and with a constant cross sectional area denoted by A. Positions in A are denoted by (x, y) coordinates; the origin (x, y) = (0, 0) defines the beam axis.

If the beam axis goes through the geometric centroid of A and if the beam is constructed of a single elastic material, certain simplifications occur. However, if multiple materials are used to construct the beam, and/or if creep/plasticity effects are present, then the simplifications may not be available. Any non-centroidal effects are incorporated directly in the equations used here.

The x,y,z coordinates denote positions fixed in the unstressed beam at the reference temperature (zero ε_T),

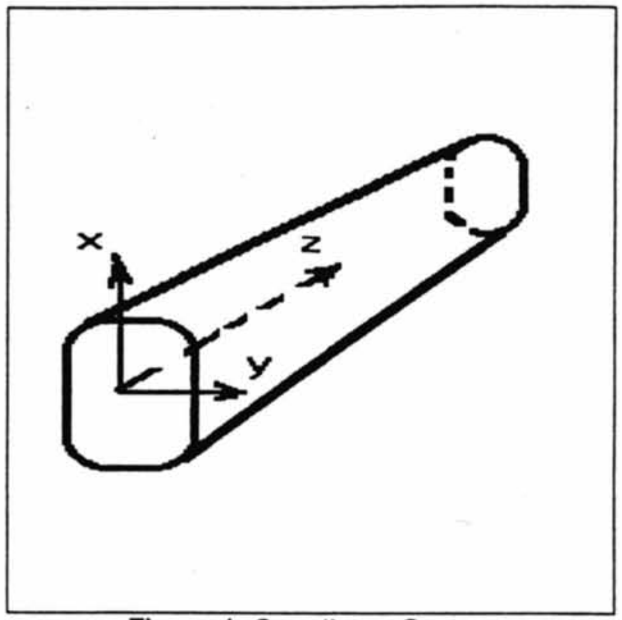


Figure 1: Coordinate System and Beam Geometry

The displacements of a point on the beam axis relative to the reference state are denoted by (u, v, w) in the (x, y, z) directions, respectively. Thus (u, v ,w) are functions of z.

The displacements of points on the beam axis are defined by giving the following quantities as a function of z:

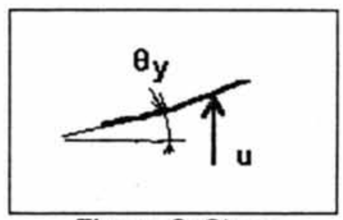


Figure 2: Slope of Beam Axis

- u — the x-direction displacement (m);
- θ_y — the slope (radians) of the beam axis (a small rotation that can be represented by a vector parallel to the y-axis); and
- k_y — the curvature (1/m) of the beam axis (the reciprocal of the radius of curvature) in the plane normal to the y-axis.

A positive sign for curvature (k_y) and for displacement (u) are indicated in Figure 3. The equality of the reciprocal of k_y and the radius of curvature is also shown schematically.

Symbols for forces and moments are also defined, as follows:

- f_x — the external load in the positive x-direction per unit axial length (N/m);
- V_x — the shear force (N) on a positive face that has a normal in the z-direction (with a positive sign illustrated in Figure 4);

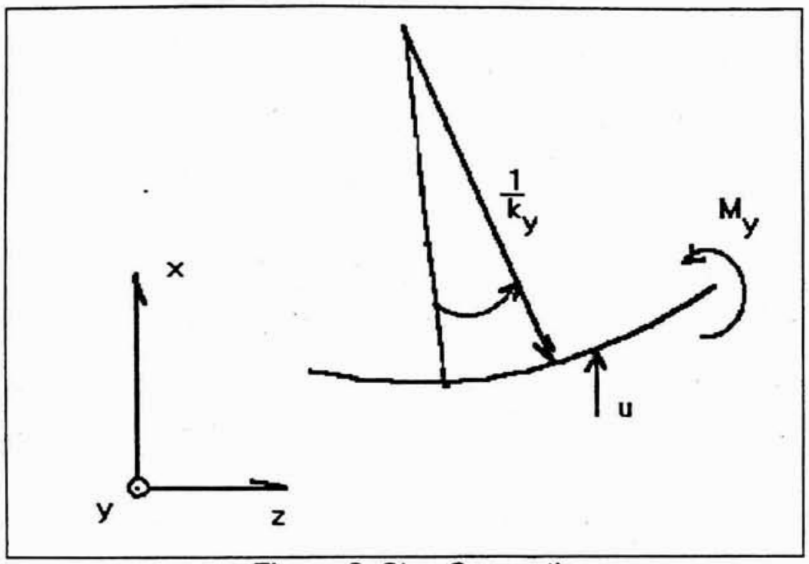


Figure 3: Sign Conventions
(a positive sign is illustrated)

M_y a moment (N·m) about an axis in the y-direction that passes through the beam axis on a positive face (a positive sign for the moment is illustrated in Figures 3 and 4); and

F_z an axial force (N) in the z-direction (acting on a positive face).

Force-Deformation (force-defo): Consider a case in which the predominant stresses are normal stresses that lie in the z-direction. Then the z-direction tensile strain is:

$$\varepsilon_z = \frac{1}{E}\sigma_z + \varepsilon_{zo} ; \qquad (1)$$

where effects of σ_x and σ_y are neglected; and where ε_{zo} is the strain in the z direction that would exist if, locally, σ_z went to zero. Contributors to ε_{zo} include thermal strain (ε_T), mechanical strain $(\varepsilon_m)_z$ from plasticity and creep, and strains $(\varepsilon_d)_z$ from other deformation mechanisms (such as radiation swelling and radiation growth):

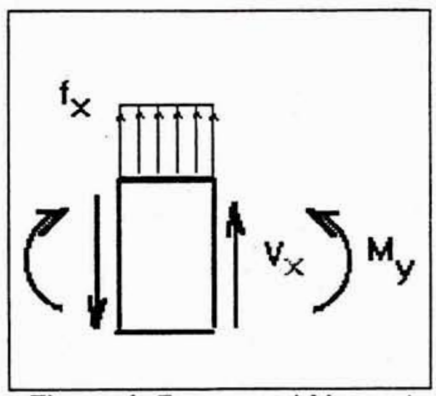


Figure 4: Forces and Moments

$$\varepsilon_{zo} = \varepsilon_T + (\varepsilon_m)_z + (\varepsilon_d)_z \ . \qquad (2)$$

The terms E, ε_T, $(\varepsilon_m)_z$, and $(\varepsilon_d)_z$ in Eqs 1 and 2 are evaluated for the local material, for the local temperature, and for the local creep/plastic/irradiation history at the particular (x, y, z) position of interest.

Deformation-Displacement (defo-displ): Beam displacements[1] are defined by giving u and w as a function of z. The additional deformation and displacement measures (slope (θ_y), curvature (k_y), and axial strain at the beam axis (ε_{za})) are obtained as follows, on a small slope and small

[1] If the beam axis is also displaced in the y-direction, then additional equations must be solved. For the present development, it is assumed that v = 0.

curvature basis (these equations also require the assumption that the beam is slender; if thick beams are encountered, then shear deformations may also be important):

$$\theta_y = \frac{du}{dz} ; \tag{3}$$

$$k_y = \frac{d\theta_y}{dz} ; \text{ and} \tag{4}$$

$$\varepsilon_{za} = \frac{dw}{dz} . \tag{5}$$

A next step is to invoke the fundamental assumption of beam theory (Popov, Ref. P-1, page 122), that "each plane section that is normal to the beam axis in the original reference state is also plane and normal to the beam axis in later deformed states." This assumption is satisfied if (in treating strains at a given axial position in the case of curvature represented by a vector parallel to the y axis):

$$\varepsilon_z = \varepsilon_{za} - k_y x . \tag{6}$$

Equilibrium (eqm): Force measures in the beam are the moment M_y, the shear force V_x, and the axial force F_z. Equations of statics are (see Figure 4):

$$\frac{dV_x}{dz} = - f_x ; \text{ and} \tag{7}$$

$$\frac{dM_y}{dz} = - V_x . \tag{8}$$

The force measures are related to the stresses in the z-direction as follows:

$$F_z = \int_A \sigma_z \, dA ; \text{ and} \tag{9}$$

$$M_y = -\int_A \sigma_z \, x \, dA . \tag{10}$$

Discussion: Consider that the quantities f_x, F_z, and ε_{zo} are known. Two boundary conditions must also be supplied at each end of the beam (to give information about u or θ_y or M_y or V_x at each location. One statement about axial displacements is also needed (e.g., w = 0 at z = 0). Note that other combinations of input may also provide well posed problems.

The solution may be thought of in the following way:

integrate Eq (7) to obtain V_x;

integrate Eq (8) to obtain M_y;

combine Eqs (1), (6), (9) and (10) to obtain a relation between M_y and k_y;

integrate the k_y relation using Eq (4) to obtain θ_y; and integrate Eq (3) to obtain u.

In performing these steps, the boundary conditions and the strain relations of Eqs (1) and (6) must be satisfied.

The equations may be simplified by noting that:

$$\int_A x \, dA = 0 \quad , \tag{11}$$

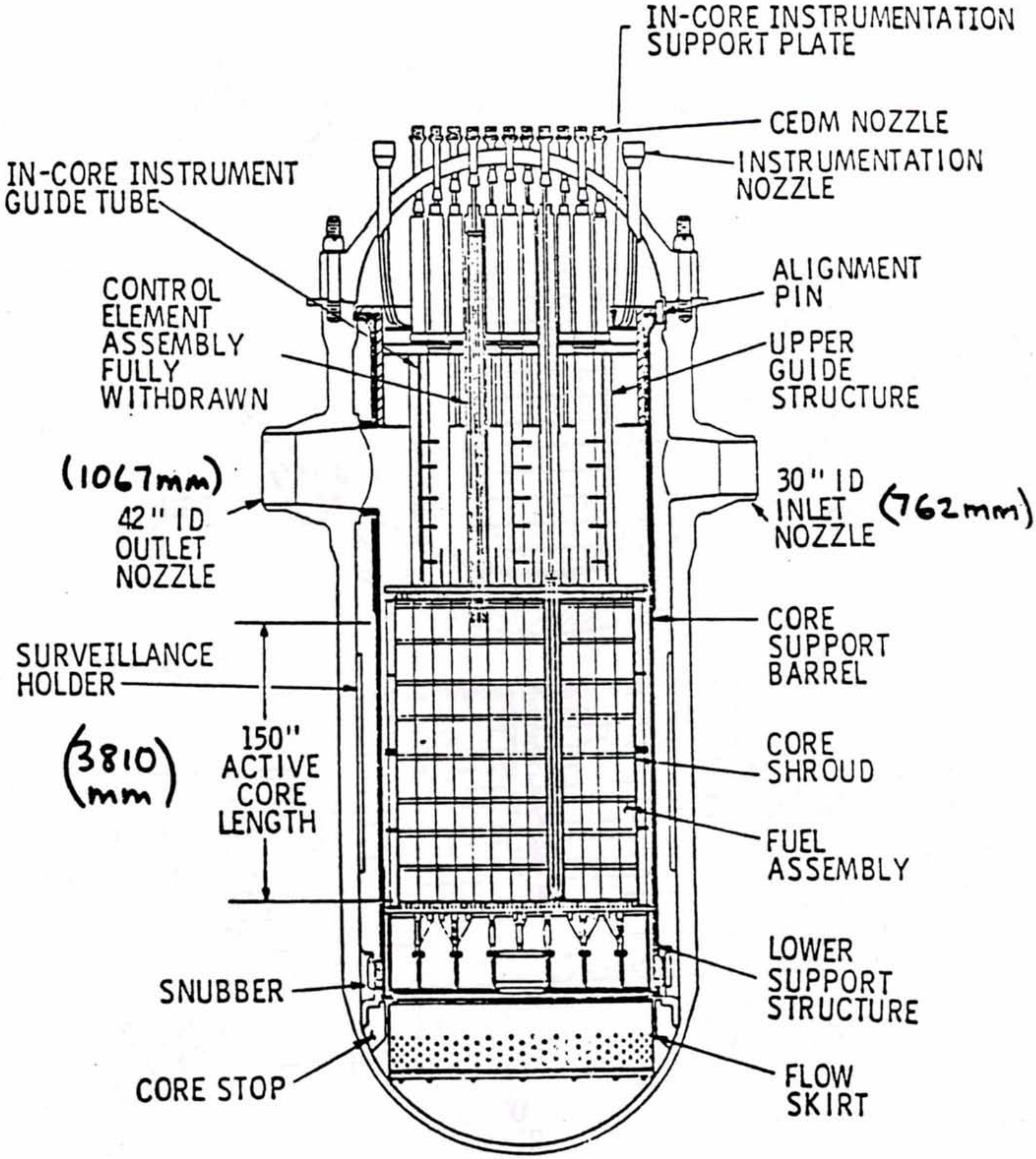
if the beam axis is centroidal with respect to the cross-sectional area and the y-axis; and

$$I_y = \int_A x^2 \, dA \tag{12}$$

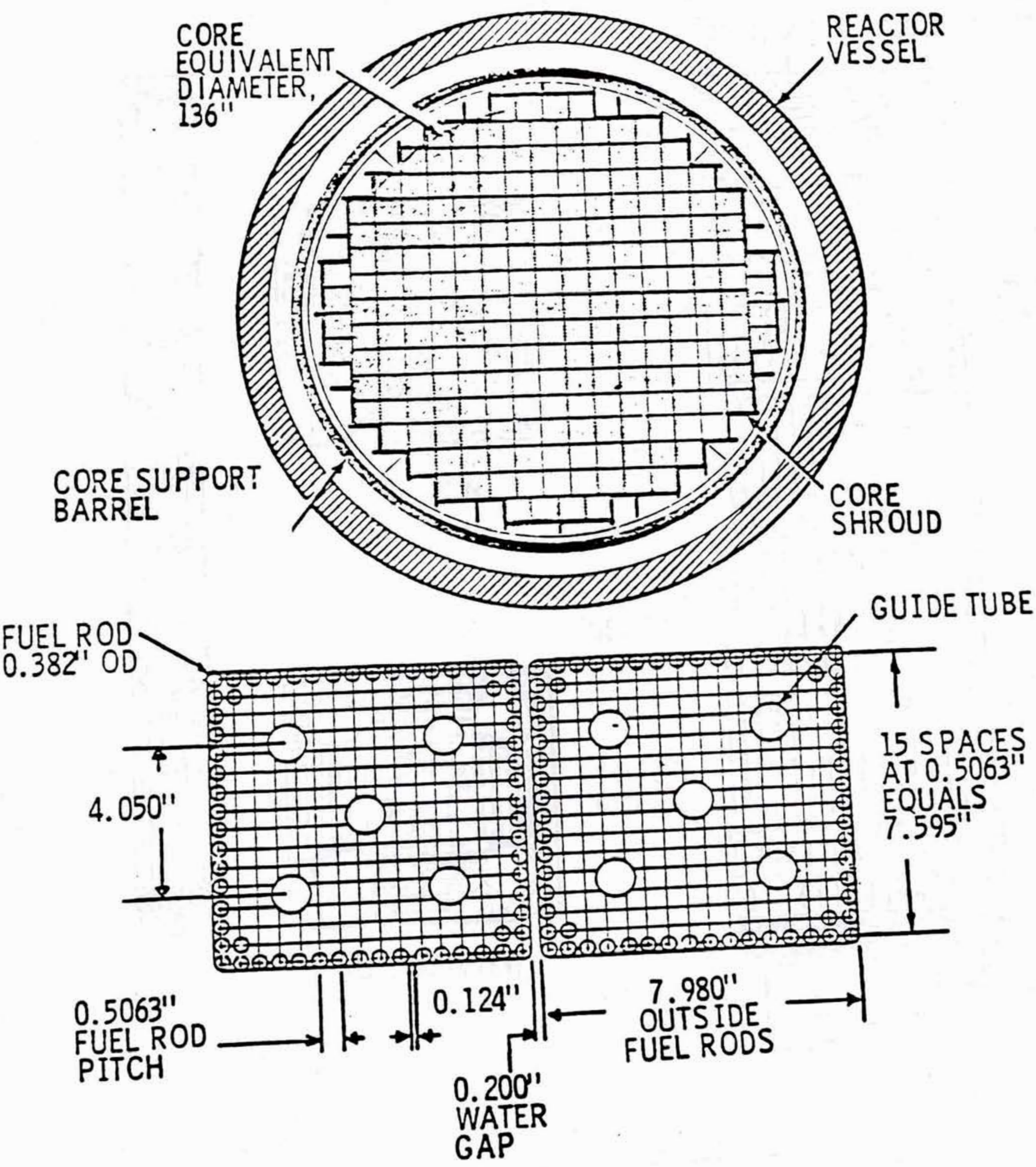
may be used to define the symbol I_y (the area moment of inertia about the y-axis).

Reference:

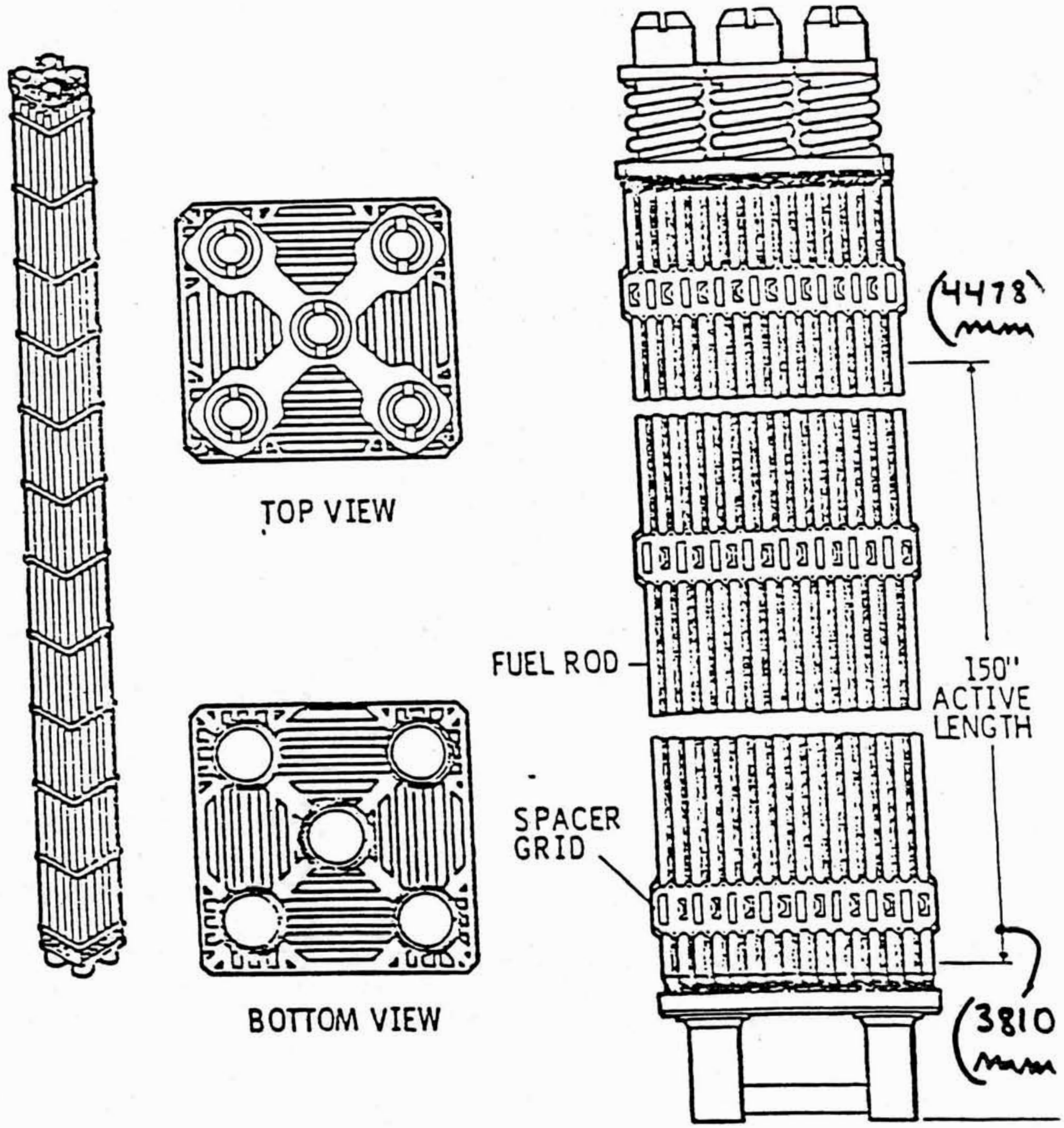
P-1 E.P. Popov, "Mechanics of Materials," 2nd ed., SI version, Prentice- Hall, Englewood Cliffs, NJ, 1978.



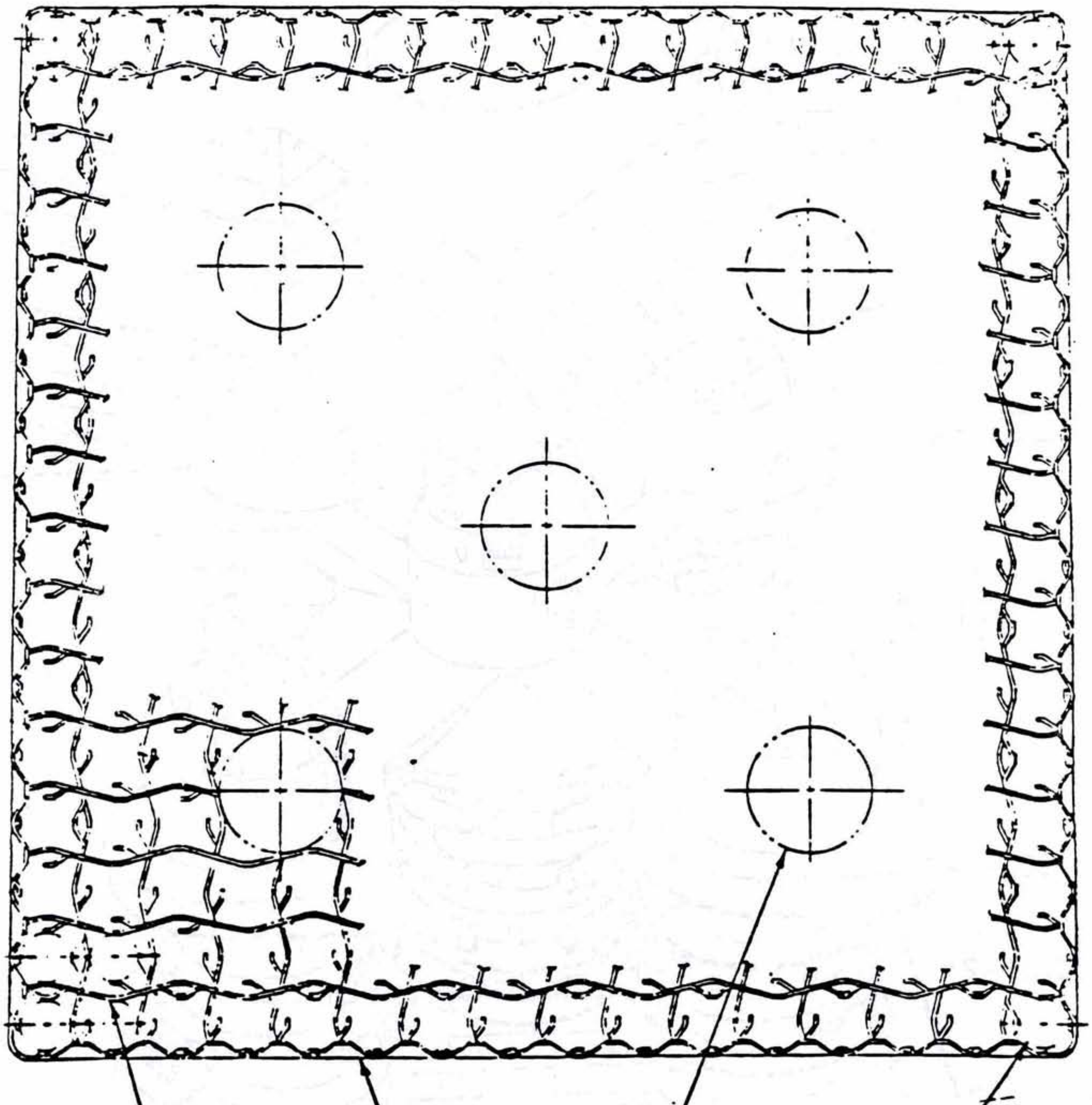
BOSTON EDISON	Reactor Vertical Arrangement	Figure



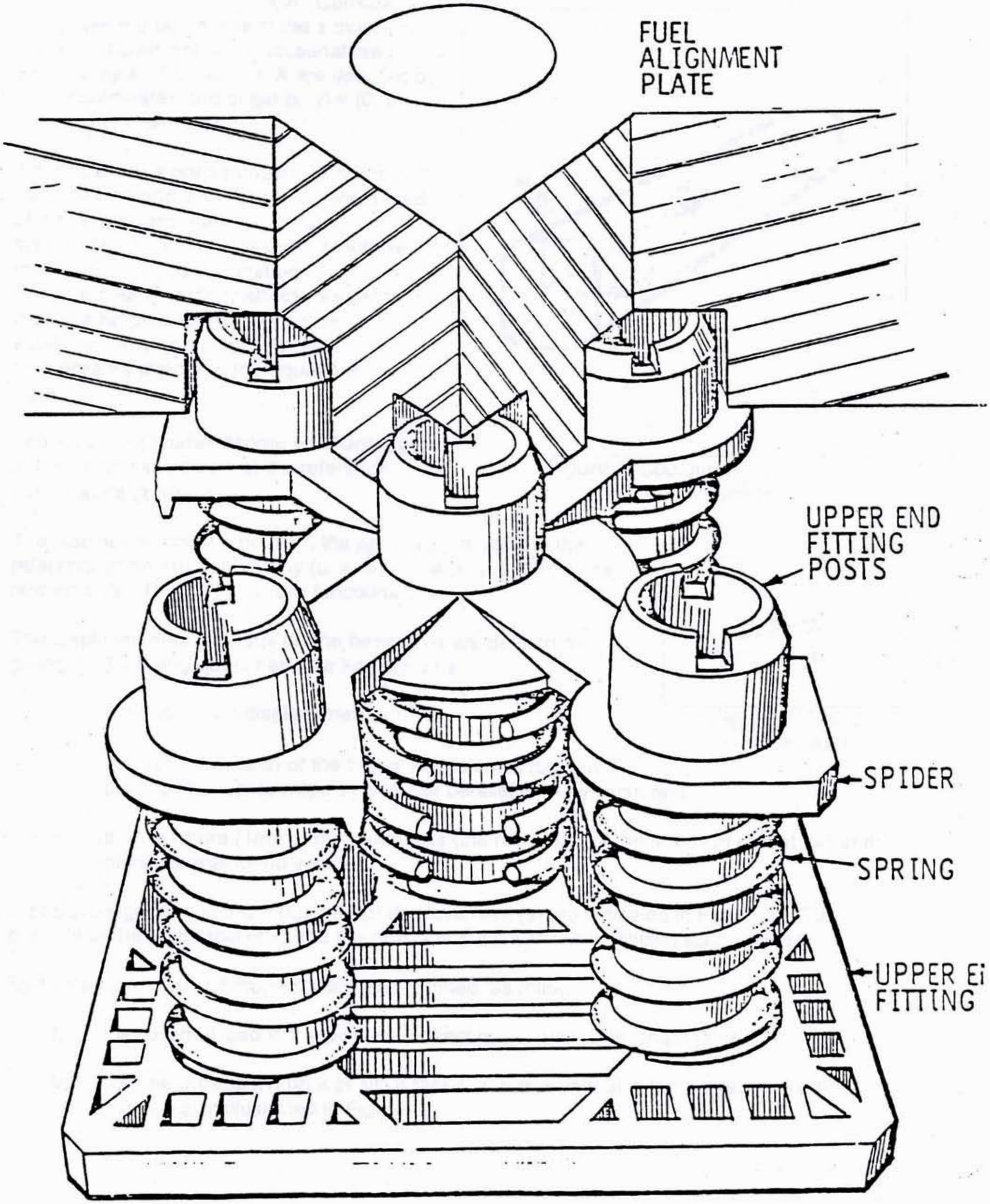
BOSTON EDISON Pilgrim Station	REACTOR CORE CROSS SECTION	Figure 4.1-2



BOSTON EDISON Pilgrim Station	FUEL ASSEMBLY



BOSTON EDISON Pilgrim Station	FUEL SPACER GRID	Figure 4.2-


FUEL ALIGNMENT PLATE
UPPER END FITTING POSTS
SPIDER
SPRING
UPPER E
FITTING