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ARTICLE A-3000

ANALYSIS OF SPHERICAL SHELLS

A-3100 INTRODUCTION

A-3110 SCOPE

(a) In this article formulas are given for stresses and deformations in spherical shells subjected to internal or external pressure.

(b) Formulas are also given for bending analysis of partial spherical shells under the action of uniformly distributed edge forces and moments.

A-3120 NOMENCLATURE AND SIGN CONVENTION

(a) The symbols and sign convention adopted in this article are defined as follows:

(1) p Uniform pressure internal or external, psi

(2) M Meridional bending moment per unit length of circumference, inch-pounds per inch

(3) H Force per unit length of circumference, perpendicular to center line of sphere, pounds per inch

(4) N Membrane force, pounds per inch

(5) Q Radial shearing force per unit of circumference, pounds per inch

(6) S Stress intensity, psi

(7) R_m Radius of mid-surface of spherical shell, inches

(8) R Inside radius, inches

(9) t Thickness of spherical shell, inches

(10) E Modulus of elasticity, psi

(11) ν Poisson's ratio

(12) D $Et^3/12(1-\nu^2)$, flexural rigidity inch-pounds

(13) β $[3(1-\nu^2)/R_m^2 t^2]^{1/4}$, inch^{-1}

(14) ϕ Meridional angle measured from center line of sphere, degrees

(15) ϕ_0 = Meridional angle of reference edge where loading is applied, degrees

(16) ϕ_L = Meridional angle of second edge, degrees

(17) α = Meridional angle measured from the reference edge, radians

(18) x = Length of arc for angle α, measured from reference edge of hemisphere, $\pi R_m \alpha/180$, inches

(19) $\lambda = \beta R_m$

(20) Y = Ratio of outside radius to inside radius

(21) Z = Ratio of outside radius to an intermediate radius

(22) U = Ratio of inside radius to an intermediate radius

(23) w = Radial displacement of mid-surface, inches

(24) δ = Lateral displacement of mid-surface, perpendicular to center line of spherical shell, inches

(25) θ = Rotation of mid-surface, radians

(26) o = As a subscript used to denote a quantity at reference edge of sphere

(27) ℓ = As a subscript used to denote meridional direction

(28) t = As a subscript used to denote circumferential direction

(29) $K_1 = 1 - \frac{1-2\nu}{2\lambda}\cot(\phi_0 - \alpha)$

(30) $K_2 = 1 - \frac{1+2\nu}{2\lambda}\cot(\phi_0 - \alpha)$

(31) $k_1 = 1 - \frac{1-2\nu}{2\lambda}\cot\phi_0$

(32) $k_2 = 1 - \frac{1+2\nu}{2\lambda}\cot\phi_0$

(33) $A = \sqrt{1 - K_1^2}$

(34) $B(\alpha) = [(1-\nu^2)(K_1 - K_2) - 2K_2]$

(35) $C(\alpha) = \sqrt{\frac{\sin\phi_0}{\sin(\phi_0 - \alpha)}}$

(36) $F(\alpha)=\sqrt{\sin(\phi_o)\sin(\phi_o)-}$

(37) $\gamma_o=\tan^{-1}(-\lambda)$

(38) $\sigma_r=$ Radial stress component, psi

(39) $\sigma_t=$ Tangential (circumferential) stress component, psi

(40) $\sigma_l=$ Longitudinal (meridional) stress component, psi

(b) **The sign convention is listed below and shown in Fig. A-3120-1 by the positive directions of the pertinent quantities:**

(p) Pressure, Positive radially outward

(δ) Lateral Displacement, Perpendicular to L of sphere, positive outward

(θ) Rotation, Positive when accompanied by an increase in the radius or curvature, as caused by a positive moment

(M), (M_o) Moment, Positive when causing tension on the inside surface

(H), (H_o) Force perpendicular to L, positive outward

(N_t), (N_l) Membrane, Force, Positive when causing tension

A-3200 STRESS INTENSITIES, BENDING ANALYSIS, DISPLACEMENTS AND EDGE LOADS

A-3210 PRINCIPAL STRESSES AND STRESS INTENSITIES RESULTING FROM INTERNAL OR EXTERNAL PRESSURE

In this Subarticle formulas are given for principal stresses and stress intensities resulting from uniformly distributed internal or external pressure in complete or partial spherical shells. The effects of discontinuities in geometry and loading are not included and should be evaluated independently. The stresses resulting from all effects must be combined by superposition.

A-3220 PRINCIPAL STRESSES AND STRESS INTENSITIES

A-3221 Principal Stresses Resulting from Internal Pressure

The principal stresses at any point in the wall of a spherical shell as a result of internal pressure are given by the following formulas:

$$\sigma_1=\sigma_t=p(Z^3+2)/2(Y^3-1) \quad \text{(a)}$$

$$\sigma_2=\sigma_l=p(Z^3+2)/2(Y^3-1) \quad \text{(b)}$$

$$\sigma_3=\sigma_r=p(1-Z^3)/(Y^3-1) \quad \text{(c)}$$

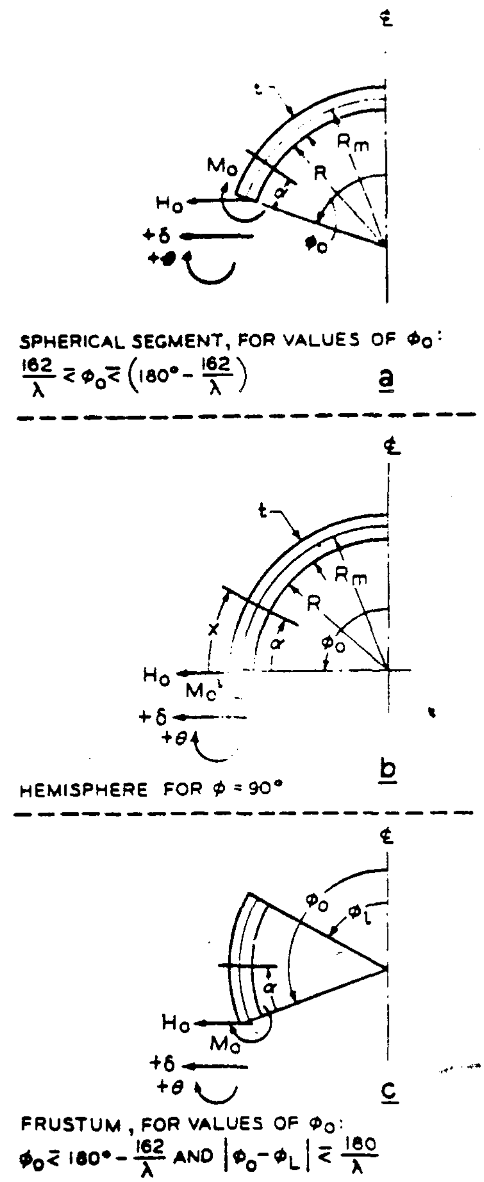

FIG. A-3120-1

A-3222 Stress Intensities Resulting From Internal Pressure

A-3222.1 General Primary-Membrane Stress Intensity. The general primary stress intensity in a spherical shell as a result of internal pressure is given by the formula:

$$S=.75\,p(Y^3+1)/(Y^3-1) \quad \text{(a)}$$

A-3222.2 Maximum Value of Primary-plus-Secondary Stress Intensity. The maximum value of t stress intensity in a spherical shell as a result of internal pressure occurs at the inside surface and is given by the formula:

$$S=1.5\,p(Y^3)/(Y^3-1) \qquad \text{(b)}$$

A-3223 Principal Stresses Resulting from External Pressure.

The principal stresses at any point in the wall of a spherical shell resulting from external pressure are given by the following formulas:

$$\sigma_1=\sigma_t=-pY^3(U^3+2)/2(Y^3-1) \qquad \text{(a)}$$

$$\sigma_2=\sigma_t=-pY^3(U^3+2)/2(Y^3-1) \qquad \text{(b)}$$

$$\sigma_3=\sigma_r=pY^3(U^3-1)/(Y^3-1) \qquad \text{(c)}$$

A-3224 Stress Intensities Resulting from External Pressure

A-3224.1 General Primary-Membrane Stress Intensity. The general primary membrane stress intensity in a spherical shell as a result of external pressure is given by the formula:

$$S_{avg.}=.75\,p(Y^3+1)/(Y^3-1) \qquad \text{(a)}$$

A-3224.2 Maximum Value of Primary-plus-Secondary Stress Intensity. The maximum value of the primary-plus-secondary stress intensity in a spherical shell as a result of external pressure occurs at the inside surface and is given by the formula:

$$S=1.5\,p(Y^3)/(Y^3-1) \qquad \text{(b)}$$

NOTE: The formulas in A-3223 and A-3224 may be used only if the applied external pressure is less than the critical pressure which would cause instability of the sphèrical shell. The value of the critical pressure must be evaluated in accordance with the rules given in NB-3133.4.

A-3230 BENDING ANALYSIS FOR UNIFORMLY DISTRIBUTED EDGE LOADS

A-3231 Scope and Limitations of Formulas Given

(a) The formulas in this subsubarticle describe the behavior of partial spherical shells of the types shown in Fig. A-3120-1 when subjected to the action of meridional bending moments, M_o (inch pounds per inch of circumference), and forces, H_o (pounds per inch of circumference), uniformly distributed at the reference edge and acting at the mean radius of the shell. The effects of all other loading must be evaluated independently and combined by superposition.

(b) The formulas listed in this paragraph become less accurate and should be used with caution when R_m/t is less than 10 and/or the opening angle limitations shown in Fig. A-3120-1 are exceeded.

A-3232 Displacement, Rotation, Moment and Membrane Force in Terms of Loading Conditions at Reference Edge

The displacement, (δ), the rotation, (θ), the bending moments, (M_t), (M_t), and the membrane forces, (N_t), (N_t), at any location of sphere are given in terms of the edge loads, M_o and H_o, by the following formulas:

$$\delta=M_o\left\{\frac{2\lambda^2}{E\,t\,k_1}F(\alpha)e^{-\lambda\alpha}\left[\cos(\lambda\alpha)-K_2\sin(\lambda\alpha)\right]\right\}$$
$$+H_o\left\{\frac{R_m\lambda}{E\,t\,k_1}A_o\sin\phi_o\,F(\alpha)e^{-\lambda\alpha}\left[\cos(\lambda\alpha+\gamma_o)-K_2\sin(\lambda\alpha+\gamma_o)\right]\right\} \qquad \text{(a)}$$

$$\theta=M_o\left\{\frac{4\lambda^3}{R_m\,E\,t\,k_1}C(\alpha)e^{-\lambda\alpha}\cos(\lambda\alpha)\right\}$$
$$+H_o\left\{\frac{2\lambda^2}{E\,t\,k_1}A_o\sin\phi_o\,C(\alpha)e^{-\lambda\alpha}\cos(\lambda\alpha+\gamma_o)\right\} \qquad \text{(b)}$$

$$M_t=M_o\left\{\frac{1}{k_1}C(\alpha)e^{-\lambda\alpha}\left[K_1\cos(\lambda\alpha)+\sin(\lambda\alpha)\right]\right\}$$
$$+H_o\left\{\frac{R_m}{2\lambda\,k_1}A_o\sin\phi_o\,C(\alpha)e^{-\lambda\alpha}\left[K_1\cos(\lambda\alpha+\gamma_o)+\sin(\lambda\alpha+\gamma_o)\right]\right\} \qquad \text{(c)}$$

$$M_t=M_o\left\{\frac{C(\alpha)}{2\,\nu\,k_1}e^{-\lambda\alpha}\left[B(\alpha)\cos(\lambda\alpha)+2\nu^2\sin(\lambda\alpha)\right]\right\}$$
$$+H_o\left\{\frac{R_m}{4\,\nu\,\lambda\,k_1}A_o\sin\phi_o\,C(\alpha)e^{-\lambda\alpha}\left[B(\alpha)\cos(\lambda\alpha+\gamma_o)+2\nu^2\sin(\lambda\alpha+\gamma_o)\right]\right\} \qquad \text{(d)}$$

$$N_t=-M_o\left\{\frac{2\lambda}{R_m\,k_1}C(\alpha)e^{-\lambda\alpha}\sin(\lambda\alpha)\cot(\phi_o-\alpha)\right\}$$
$$-H_o\left\{\frac{1}{k_1}A_o\cot(\phi_o-\alpha)\,C(\alpha)e^{-\lambda\alpha}\sin(\lambda\alpha+\gamma_o)\right\} \qquad \text{(e)}$$

$$N_t=M_o\left\{\frac{2\lambda^2}{R_m\,k_1}C(\alpha)e^{-\lambda\alpha}\left[\cos(\lambda\alpha)-\left(\frac{K_1+K_2}{2}\right)\sin(\lambda\alpha)\right]\right\}$$
$$+H_o\left\{\frac{\lambda}{k_1}A_o\sin\phi_o\,C(\alpha)e^{-\lambda\alpha}\left[\cos(\lambda\alpha+\gamma_o)-\left(\frac{K_1+K_2}{2}\right)\sin(\lambda\alpha+\gamma_o)\right]\right\} \qquad \text{(f)}$$

A-3233 Displacement and Rotation of Reference Edge in Terms of Loading Conditions at Reference Edge

A-3233.1 At the Reference Edge Where $\alpha=0$, and $\phi=\phi_o$. The formulas for the displacement and rotation (A-3232) simplify to those given below:

$$\delta_o=M_o\frac{2\lambda^2\sin\phi_o}{E\,t\,k_1}+H_o\frac{R_m\,\lambda\sin^2\phi_o}{E\,t}\left(\frac{1}{k_1}+k_2\right)\quad\text{(a)}$$

$$\theta_o=M_o\frac{4\lambda^3}{R_m\,E\,t\,k_1}+H_o\frac{2\lambda^2\sin\phi_o}{E\,t\,k_1}\quad\text{(b)}$$

A-3233.2 When Shell Is A Full Hemisphere. In the case where the shell under consideration is a full hemisphere the formulas given in (a) and (b) above reduce to those given below:

$$\delta_o=M_o\frac{2\lambda^2}{E\,t}+H_o\frac{2R_m\lambda}{E\,t}\quad\text{(a)}$$

$$\theta_o=M_o\frac{4\lambda^3}{R_m\,E\,t}+H_o\frac{2\lambda^2}{E\,t}\quad\text{(b)}$$

A-3234 Principal Stresses in Spherical Shells Resulting from Edge Loads

The principal stresses at the inside and outside surfaces of a spherical shell at any location, resulting from edge loads, M_o and H_o, are given by the following formulas:

$$\sigma_1=\sigma_l(\alpha)=\frac{N_l}{t}\pm\frac{6M_l}{t^2}\quad\text{(a)}$$

$$\sigma_2=\sigma_t(\alpha)=\frac{N_t}{t}\pm\frac{6M_t}{t^2}\quad\text{(b)}$$

$$\sigma_3=\sigma_r(\alpha)=0\quad\text{(c)}$$

In these formulas where terms are preceded by a double sign, $\pm$, the upper sign refers to the inside surface of the shell and the lower sign refers to the outside surface.

A-3240 ALTERNATE BENDING ANALYSIS OF A HEMISPHERICAL SHELL SUBJECTED TO UNIFORMLY DISTRIBUTED EDGE LOADS

A-3241 Nature of Formulas Given

If a less exacting but more expedient analysis of hemispherical shells is required, formulas derived for cylindrical shells may be used in a modified form. The formulas listed in A-3242 describe the behavior of a hemispherical shell as approximated by a cylindrical shell of the same radius and thickness when subjected to the action of uniformly distributed edge loads, M_o and H_o, at $\alpha=0$, $x=0$, and $\phi_o=90°$.

A-3242 Displacement, Rotation, Moment and Shear Forces in Terms of Loading Conditions at Edge

$$\delta_o=H_o/2\beta^3D+M_o/2\beta^2D\quad\text{(a)}$$

$$\theta_o=H_o/2\beta^2D+M_o/\beta D\quad\text{(b)}$$

$$\delta(x)=\frac{H_o\sin^2\phi}{2\beta^3D}f_1(\beta x)+\frac{M_o\sin\phi}{2\beta^2D}f_2(\beta x)\quad\text{(c)}$$

$$\theta(x)=\frac{H_o\sin\phi}{2\beta^2D}f_2(\beta x)+\frac{M_o}{\beta D}f_1(\beta x)\quad\text{(d)}$$

$$M(x)=\frac{H_o\sin\phi}{\beta}f_4(\beta x)+M_of_3(\beta x)\quad\text{(e)}$$

$$Q(x)=H_o\sin\phi\,f_2(\beta x)-2\beta\,M_of_4(\beta x)\quad\text{(f)}$$

Where f_1, f_2, f_3 and f_4 are defined in Article A-2000, Analysis of Cylindrical Shells, and

$$x=\alpha R_m=\frac{\pi R_m}{180°}(90°-\phi)$$

A-3243 Principal Stresses in a Hemispherical Shell Due to Edge Loads

The principal stresses in a hemispherical shell due to edge loads, (M_o) and (H_o), at the inside and outside surfaces of a hemispherical shell at any meridional location, are given by the formulas:

$$\sigma_1=\sigma_l(x)=\pm 6M(x)/t^2\quad\text{(a)}$$

$$\sigma_2=\sigma_t(x)=E\,\delta(x)/R_m\pm\nu 6M(x)/t^2\quad\text{(b)}$$

$$\sigma_3=\sigma_r=0\quad\text{(c)}$$

In these formulas where terms are preceded by a double sign, $\pm$, the upper sign refers to the inside surface of the hemisphere and the lower sign refers to the outside surface.