Chapter 8
Energy Transport

Background

- Chapter 5 dealt with how the neutron population varies in time
- Chapter 6 and 7 dealt with the spatial distribution of neutrons in the core
  - It was determine that a critical reactor could operate at any power level and that the equilibrium would hold
    - Not entirely true!
    - At high power levels, temperature changes will create important transients

Objectives

- Find simple expressions to approximate fuel temperature and coolant temperature in a reactor
  - Steady-state temperatures
  - Transients
    - Determine temperature effect on reactivity

Core Power Density

$$\bar{P}''' = P/V$$

- P''' is the power density
- P is the total core power
- V is the core volume

Power Peaking Factor

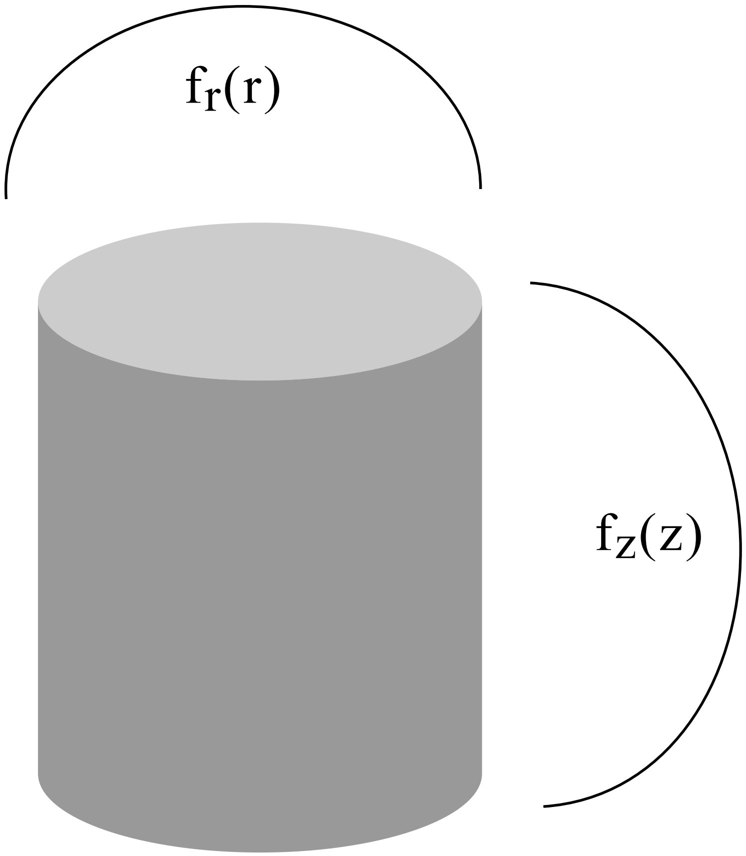
$$F_q = P'''_{\max} / \bar{P}'''$$

- F_q is the power peaking factor
- P'''_{max} is the maximum power density in the core

$$P = \frac{P'''_{\max}}{F_q} V$$

- We can combine the two previous equations and find the above relation
- Cores are designed to operate at a given power P
  - Maximizing the ratio P'''max / Fq will allow for smaller reactor design at given power P
    - Reduces construction cost
  - Main job of reactor physicist is to maximize this ratio
    - Control rods
    - New designs
    - Varying enrichment
    - …

- P''' max depends primarily on material properties
  - Temperature and pressure that can be tolerated by fuel, coolant, structure
- Fq can be lowered by playing with enrichment loading, position of control rods, poisons, reflector, …



$f_q(r,z) = f_r(r) f_z(z)$

- For a uniform bare reactor
  - $F_q = F_r F_z$
- From chapter 7, we've seen the solution for such a reactor
  - $F_r = 2.32$
  - $F_z = 1.57$
- More complicated geometries will have higher peaking due to local variations

Simple heat transfer on fuel element

- Define q' has the linear heat rate (kW/m)
  - Thermal power produced per unit length
  - $P''' = q' / A_{cell}$ where A_{cell} is the area of the fuel element

$$q' = PF_q A_{cell} / V = PF_q / NH$$

N: Number of fuel pins

H: Height of the core

Steady-state temperatures

- Temperature drop from fuel to coolant is proportional to linear heat rate

$$T_{fe}(r, z) - T_c(r,z) = R'_{fe} q'(r,z)$$

  - R'_{fe} is the fuel element thermal resistance, details of which are found in Appendix D
    - function of the thermal$_{con}$ ductivity of the fuel and the cladding as well as the heat transfer coefficient

- If we average over the volume

$$\overline{T}_f - \overline{T}_c = R_f P$$

where

$$R_f = \frac{1}{\mathrm{NH}} R'_{fe}$$

It should be noted that when averaging q' using the previous relation q' = PF_q / NH, we must note that the average core peaking factor is 1.

We know Rf and P, but we need more information to evalute Tf and Tc

- Coolant heat balance

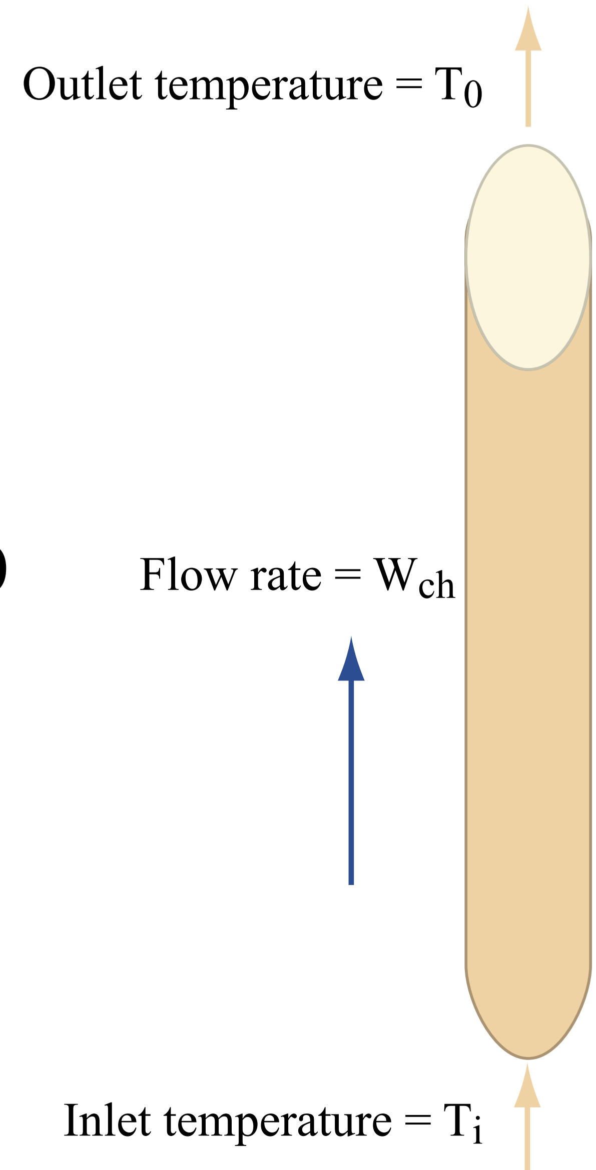
$$W_{ch}c_p[T_0(r) - T_i] = \int_{-H/2}^{H/2} q'(r, z')dz'$$

  - Replacing q' and solving for T_0

$$T_0(r) = \frac{1}{Wc_p}\,\frac{P}{NH} f_r(r) \int_{-H/2}^{H/2} f_z(z)dz + T_i$$

  - Define total core flow rate

$$W = NW_{ch}$$



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- We get

$$T_0(r) = \frac{1}{Wc_p} P f_r(r) + T_i$$

- If we average radially

$$\overline{T}_0 = \frac{1}{Wc_p} P + T_i$$

- Average coolant temperature

$$\overline{T}_\mathrm{c} = \frac{1}{2}(\overline{T}_o + T_i)$$

- We can than get an expression for the coolant temperature

$$\overline{T}_\mathrm{c} = \frac{1}{2Wc_p} P + T_i$$

- And we can replace it in our previous expression

$$\overline{T}_f = \left(R_f + \frac{1}{2Wc_p}\right) P + T_i$$

Fuel Thermal transients

- If cooling is turned off, we can approximate that the fuel will heat up adiabatically

$$M_f c_f \frac{d}{dt}\bar{T}_f(t) = P(t) - \frac{1}{R_f}[\bar{T}_f(t) - \bar{T}_C(\mathrm{t})]$$

M_f is the total fuel mass

c_f is the fuel specific heat

- Bounding cases
  - Steady-state d/dt =0 $P = (\overline{T}_f - \overline{T}_c)/R_f$
  - No cooling (R_f tends to infinity) $M_f c_f \frac{d}{dt}\overline{T}_f(t) = P(t)$
    - All the power stays in the fuel, fuel temperature increases and can eventually lead to melting
- More convenient form

$$\frac{d}{dt}\overline{T}_f(t) = \frac{1}{M_f c_f} P(t) - \frac{1}{\tau}[\overline{T}_f(t) - \overline{T}_c(t)]$$

τ is the core thermal time constant

Coolant thermal transient

- Consevation equation in the coolant

$$M_c c_p \frac{d}{dt} \overline{T}_c(t) = \frac{1}{R_f} [\overline{T}_f(t) - \overline{T}_c(t)] - 2Wc_p [\overline{T}_c(t) - T_i]$$

Heating term $\frac{1}{R_f} [\overline{T}_f(t) - \overline{T}_c(t)]$

Cooling term $2Wc_p [\overline{T}_c(t) - T_i]$

- We can rewrite has

$$\frac{d}{dt}\bar{T}_c(t) = \frac{1}{\tau'}[\bar{T}_f(t) - \bar{T}_c(t)] - \frac{1}{\tau''}[\bar{T}_c(t) - T_i]$$

where $\tau' = \dfrac{M_c c_p}{M_f c_f}\tau$ and $\tau'' = \dfrac{M_c c_p}{2Wc_p}$:

- Coolant usually follows fuel surface transient quite rapidly
  - We can ignore the energy storage term of the coolant equation

$$\bar{T}_c(t) = \frac{1}{1 + 2R_f W c_p}[2R_f W c_p T_i + T_f(t)]$$

- Combining with the fuel transient expression

$$\frac{d}{dt}\,\overline{T}_f(t) = \frac{1}{M_f c_f}P(t) - \frac{1}{\tilde{\tau}}\,[\overline{T}_f(t) - T_i\,]$$

Chapter 9
Reactivity Feedback

Background

- Temperature increase will create feedback mechanisms in the reactor
  - Doppler broadening
  - Thermal expansion
  - Density changes which will induce spectral shifts

- These changes will impact the reactivity, thus causing transients

Reactivity Coefficients

- Dynamic reactivity was defined by

$$\rho(t) = \frac{k(t) - 1}{k(t)}$$

- We can relate a change in reactivity to a change in *k*

$$dp = dk/k^2 \approx dk/k = d(\ln k)$$

- The advantage is that we change a multiplication of terms into a sum of terms

Fuel Temperature Coefficient

- Doppler broadening of the resonance capture cross-section of U-238 is the dominant effect in LWR reactors
  - Lots of U-238 (red) present
  - Similar effect with Th-232 (green)

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Fuel Temperature Coefficient

- Effect is felt in resonance escape probability (p)
- No effect on ε,because …
- Minor effect on η and f
  - Especially in the presence of Pu-239
- Doppler effect arises from the temperature dependence of the cross-section on the relative speed between neutron and nucleus
  - Resonances are smeared in energy has temperature increases.
  - Thermal resonances are more important

Fuel Temperature Coefficient

- We can approximate it by

$$\alpha_f = \frac{1}{k}\frac{\partial k}{\partial \bar{\bar{T}}_f} \approx \frac{1}{p}\frac{\partial p}{\partial \bar{\bar{T}}_f}$$

- You can evaluate it using formulas from Chapter 4 used to determine p (see book)
- Or, you can also run simulations at different fuel temperatures and compare the estimate of the eigenvalue

Moderator Temperature Coefficient

- We seek to evaluate

$$\alpha_m = \frac{1}{k}\frac{\partial k}{\partial \overline{T}_m}$$

- The biggest impact of the moderator temperature comes from associated changes in density
  - As temperature increases, moderator (and coolant) will see a decrease in their density
    - Less water molecules, means less moderation, leading to a spectral shift

Moderator Temperature Coefficient

- Decrease in slowing down efficiency will lead to an increase in resonance absorption
  - Value of p will decrease
- Lower coolant density will also have an impact on the thermal utilization (f)
  - Value of f will increase
- Fast fission will increase slightly, but effect is negligible
- The combine effect is usually negative, but in some reactors with solid moderators (i.e. graphite), the coefficient might be positive over certain temperature ranges.

Coolant Void Reactivity Coefficient

- In LWRs and BWRs, this coefficient is always negative
  - Coolant and moderator are the same, thus losing the coolant also implies loosing all the moderation
- In CANDU and RBMK, this coefficient is positive
  - Loosing the coolant as very little impact on the moderation
    - Causes slight increase in fast fission
    - Causes slight increase in resonance escape probability
  - Before Chernobyl, void reactivity coefficient of RBMK was 4.7 beta, after re-design it was lowered to 0.7 beta
  - CANDU have a very small positive reactivity coefficient that can be controlled easily

Fast Reactor Coefficients

- Leakage plays a more important role in fast reactor transients
  - Decreasing density will make the spectrum harder
    - Larger value of η, thus increase in k
  - Migration length would also increase
    - More leakage, thus decreasing k
  - Overall effect is usually positive
- Doppler effect is smaller in magnitude
  - Thermal resonances are more affected

Isothermal Temperature Coefficient

- In many reactors, the entire core is brought very slowly from room temperature to the operating inlet coolant temperature
  - Reactor at low power
  - External heat source
  - Decay heat
- Reasonable approximation is to assume that the core behaves isothermally

  $T_f = T_c = T_i$
- We can thus define the isothermal temperature coefficient

$$\alpha_T \equiv \frac{d\rho_{fb}}{d\bar{\bar{T}}} = \frac{1}{k}\frac{\partial k}{\partial \bar{\bar{T}}_f} + \frac{1}{k}\frac{\partial k}{\partial \bar{\bar{T}}_c} \qquad \alpha_T = \alpha_f + \alpha_c$$

Temperature Defect

- This coefficient allows us to estimate the amount of reactivity needed to maintain criticality at high temperature (hot zero power)
- This reactivity is obtained by integrating the isothermal temperature coefficient from room temperature to hot temperature

$$D_T = \int_{T_r}^{T_i} \alpha_T(\overline{T})\, d\overline{T}$$

Power coefficient

- A far more useful coefficient, it takes into account impact of temperature changes when reactor is operating at full power

$$\alpha_P \equiv \frac{d\rho_{fb}}{dP} = \frac{1}{k}\frac{\partial k}{\partial \overline{T}_f}\frac{d\overline{T}_f}{dP} + \frac{1}{k}\frac{\partial k}{\partial \overline{T}_c}\frac{d\overline{T}_c}{dP}$$

- If we assume that power changes are slow compared to the time required for heat removal, we can use the steady-state temperature profiles from Chapter 8 and derive them with respect to Power

$$\bar{T}_c = \frac{1}{2Wc_p}P + T_i \qquad \frac{d\bar{T}_c}{dP} = \frac{1}{2Wc_p}$$

$$\bar{T}_f = \left(R_f + \frac{1}{2Wc_p}\right)P + T_i \qquad \frac{d\bar{T}_f}{dP} = R_f + \frac{1}{2Wc_p}$$

Power coefficient

- The power coefficient is thus expressed in terms of both the fuel coefficient and the moderator coefficient

$$\alpha_P = \left(R_f + \frac{1}{2Wc_p}\right)\frac{1}{k}\frac{\partial k}{\partial \bar{T}_f} + \frac{1}{2Wc_p}\frac{1}{k}\frac{\partial k}{\partial \bar{T}_c}$$

$$\alpha_P = R_f\alpha_f + (2Wc_p)^{-1}(\alpha_f + \alpha_c)$$

- Thus, as power is increased, positive reactivity is required to overcome negative coefficients and maintain criticality

Power Defect

- As power increases to its operating level, additional negative reactivity is introduced by an increase in temperature
- We can evaluate the power defect by the following

$$D_p = \int_{T_i}^{\bar{T}_f(p)} \alpha_f(\bar{T}_f)\, d\bar{T}_f + \int_{T_i}^{\bar{T}_c(p)} \alpha_e(\bar{T}_c)\, d\bar{T}_c$$

where $T_f(P)$ and $T_c(P)$ are the fuel and coolant temperatures at power P

Typical values

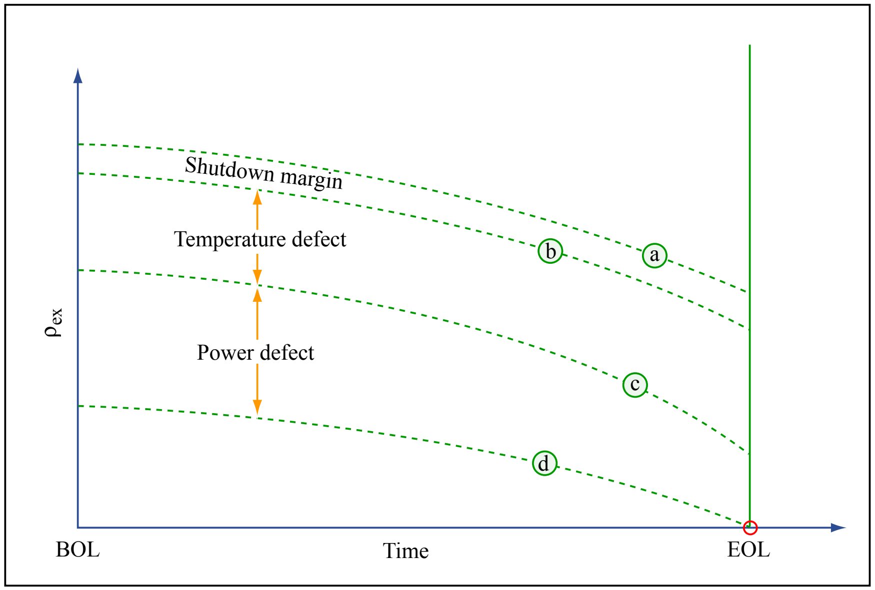
- Temperature Defect
- Power Defect
- Good exercise: Lewis 9.4

Excess Reactivity

- Defined as the value of rho if all control poisons and rods were removed from the core
  - Large excess reactivity are avoided because they need lots of poison to compensate at BOC (beginning of cycle) and require extra care
  - Creates dangerous scenarios (e.g. high worth control rods become a problem if ejected)
  - Strict limits are thus placed on excess reactivity and on the reactivity limits of control devices
    - Large amount of small control rods

- Negative temperature coefficients are nice from a stability and safety point of view, large negative values can create excess reactivity problems
- Plot depicts
  - Cold shutdown (a)
  - Cold critical (b)
  - Hot zero power critical (c)
  - Full power (d)

- Temperature feedback causes excess reactivity to decrease
  - Need to pull out control rods
- If you shutdown, temperature decreases and excess reactivity is increased
  - Need to insert control rods as you reduce power



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Shutdown Margin

- A minimum shutdown margin is imposed by the NRC
  – Reactivity required to shutdown the reactor no matter in which condition (cold critical is the one with the most excess reactivity)
  – The stuck rod criteria is usually applied
  – Normally 1-5% of excess reactivity
- Going from curve a to b removes the excess margins to get to cold critical
- As the core is heated, the excess reactivity curve goes from b to c, with the difference being the temperature defect
  – Slow temperature increase to reduce mechanical stresses on pipes and pressure vessel
- As the reactor goes up in power, we approach curve d
  – Remaining excess reactivity is what allows the core to operate for a given cycle
  – Typical LWR cycle 1-2 years

- Core designers try to predict excess reactivity curves
  - Schedule outages
  - Prepare reloading
    - Cores are usually reloaded in 3-4 batches, thus in a PWR you replace about 60 assemblies at each cycle
    - Typical assemblies will thus stay in the core for 3 cycles or 4.5 years
    - Fuel is then sent to spent fuel pools for at least 5 years
    - Pool configuration is important to avoid criticality accidents
    - When pool is full, oldest spent fuel elements are put in dry casks

- If they fall short on reactivity, they can reduce power to reduce temperature and increase excess reactivity
- If they under predict the excess reactivity, it indicates that they loaded more fresh fuel bundles than they needed
  - Reactor is still shutdown on schedule due to mobilization of workforce
  - $$$
- Outages usually last 3-4 weeks

Reactor Transients

- If rapid changes of power occur, steady-state temperatures cannot be used
  - Rod ejection
  - Loss of coolant
  - Loss of flow
- We can develop a simple reactor dynamics model based on the kinetics relation, and the temperature transient models

- Power $$\frac{d}{dt}P(t) = \frac{[\rho(t) - \beta]}{\Lambda}P(t) + \sum_i \lambda_i \tilde{C}_i(t)$$
- Precursors $$\frac{d}{dt}\tilde{C}_i(t) = \frac{\beta_i}{\Lambda}P(t) - \lambda_i \tilde{C}_i(t) \qquad i = 1, 2, 3, 4, 5, 6$$
- Fuel temperature $$\frac{d}{dt}\overline{T}_f(t) = \frac{1}{M_f c_f}P(t) - \frac{1}{\tilde{\tau}}[\overline{T}_f(t) - T_i]$$
- Coolant temperature $$\overline{T}_c(t) = T_i + \frac{1}{2R_f W c_p}T_f(t)$$

Feedback effects

- The reactivity will also have to include the temperature feedback effects

$$\rho(t) = \rho_i(t) - |\alpha_f|\,[\bar{T}_f(t) - \bar{T}_f(0)] - |\alpha_c|\,[\bar{T}_c(t) - \bar{T}_c(0)]$$

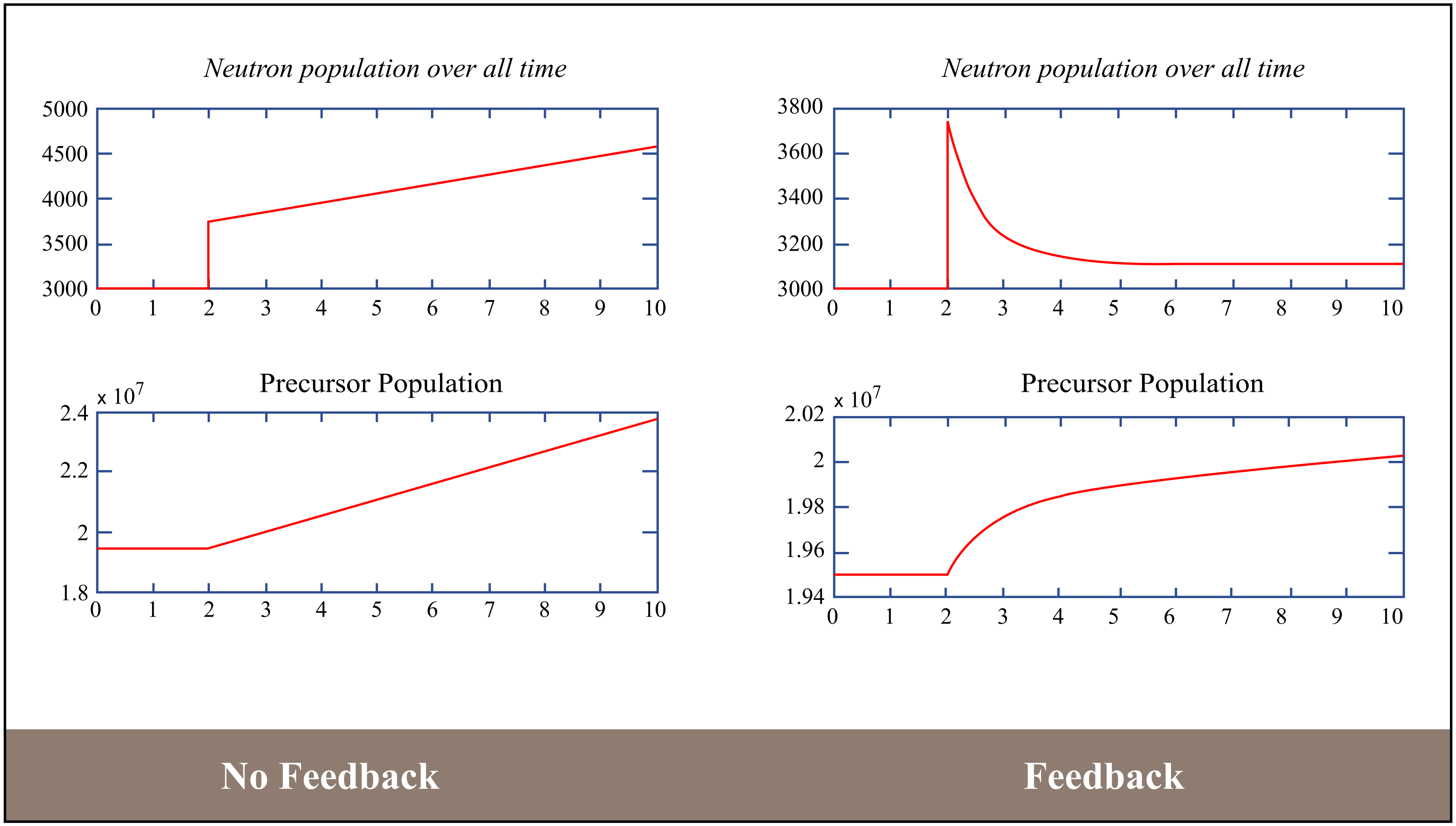
- If the reactor is initially critical at power P_0 we can evaluate the temperatures and precursor concentrations using the steady-state relations

Demo – Step insertion

- Beta = 0.0065

- Full power = 3000 MWth
  - T inlet = 300 C
  - T fuel = 1142 C

Step of 0.2$ - Full Power



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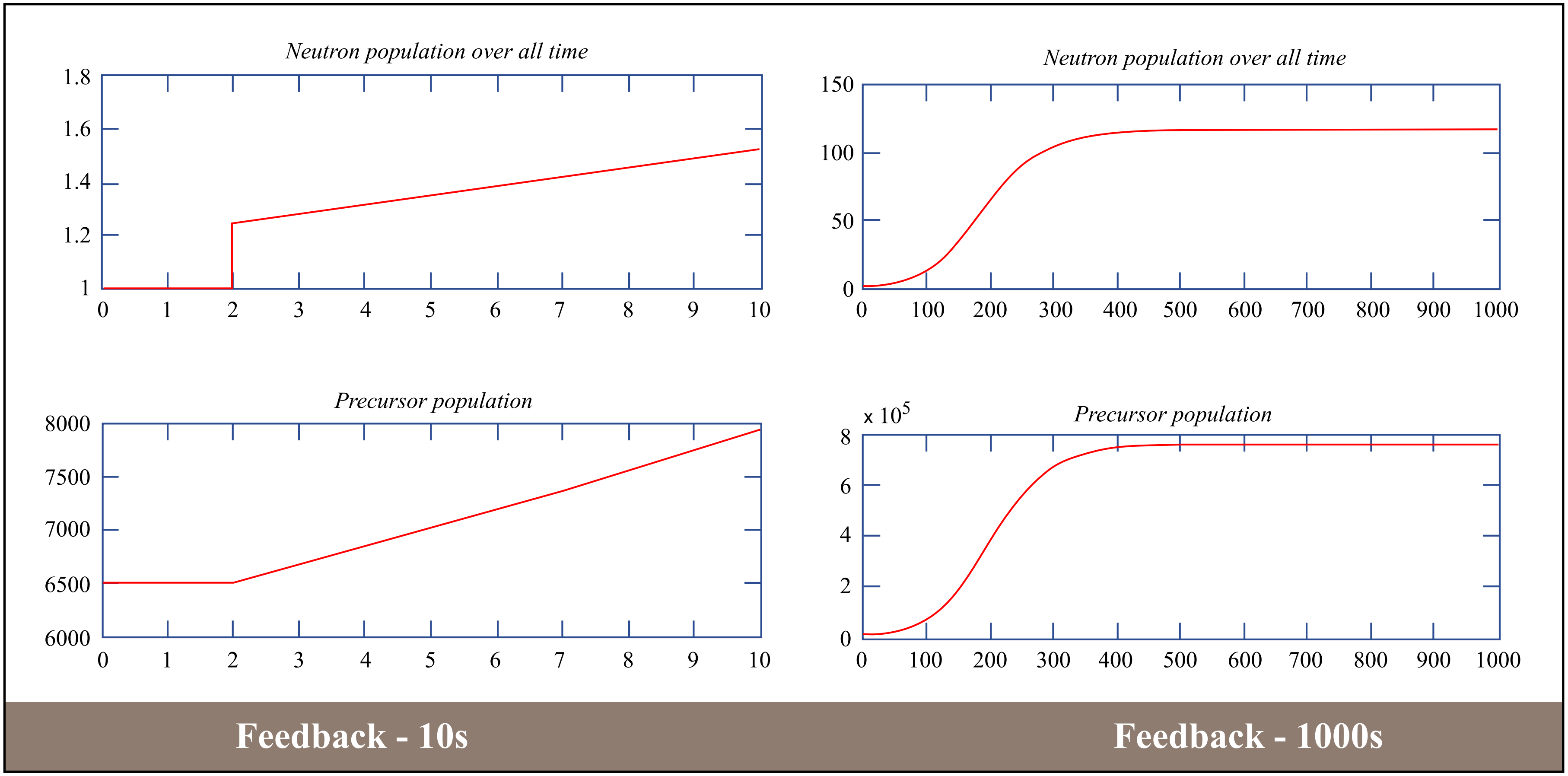
- T fuel at 10 seconds = 1173 C
- Prompt jump brings power to 3700 MWth
  - Stabilizes to 3100 MWth with feedback

Step of 0.2$ - Low Power (1 MWth)



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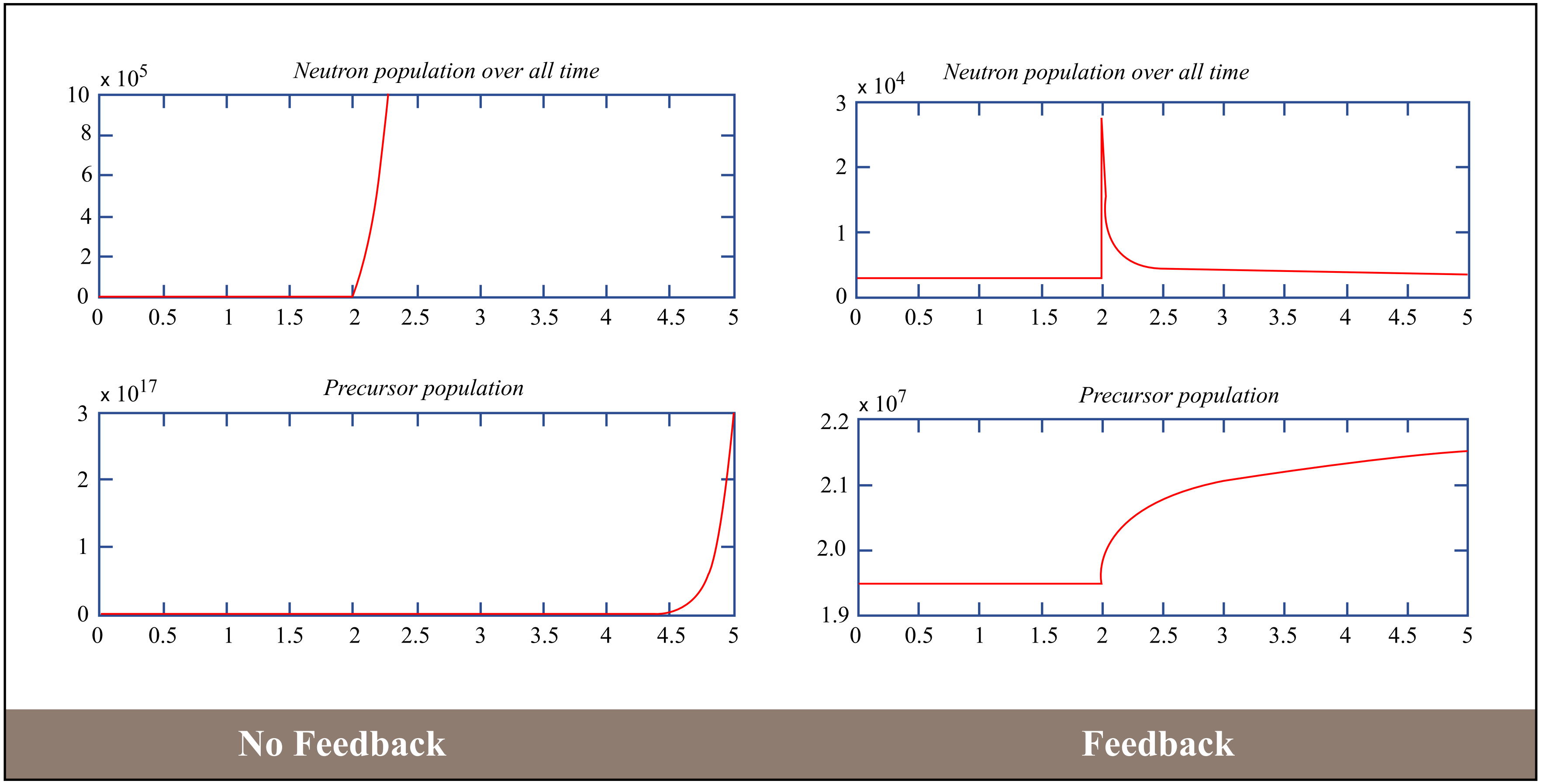
- Fuel temperature eventually reaches 333 C
- Power eventually stabilizes to 120 MWth

Step of 1$ at Full power



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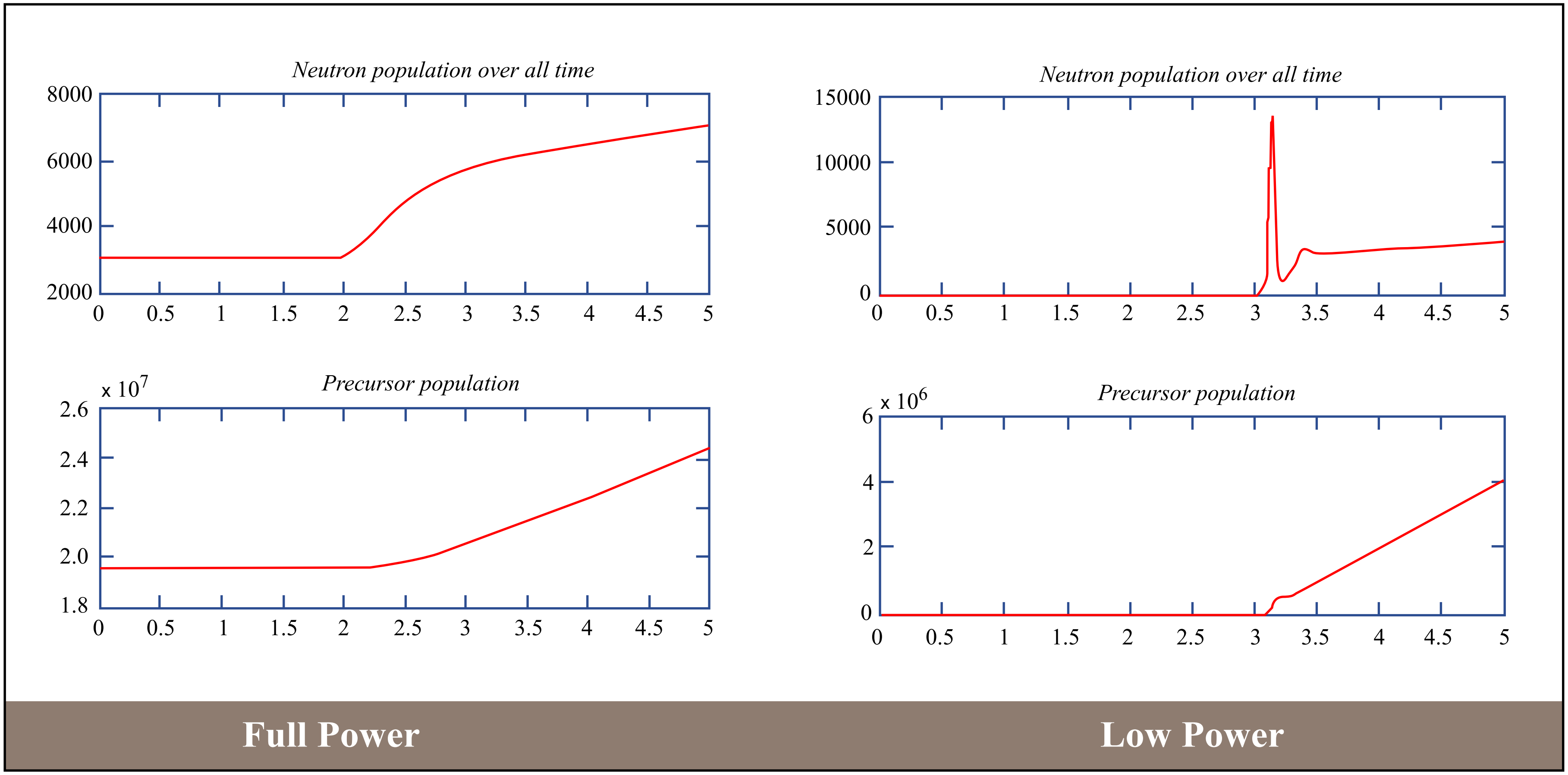
- Power spikes to 27500 MWth
- Stabilizes to 3567 MWth
- Fuel temperature reaches 1293 C

Ramp insertion 1$/s with Feedback



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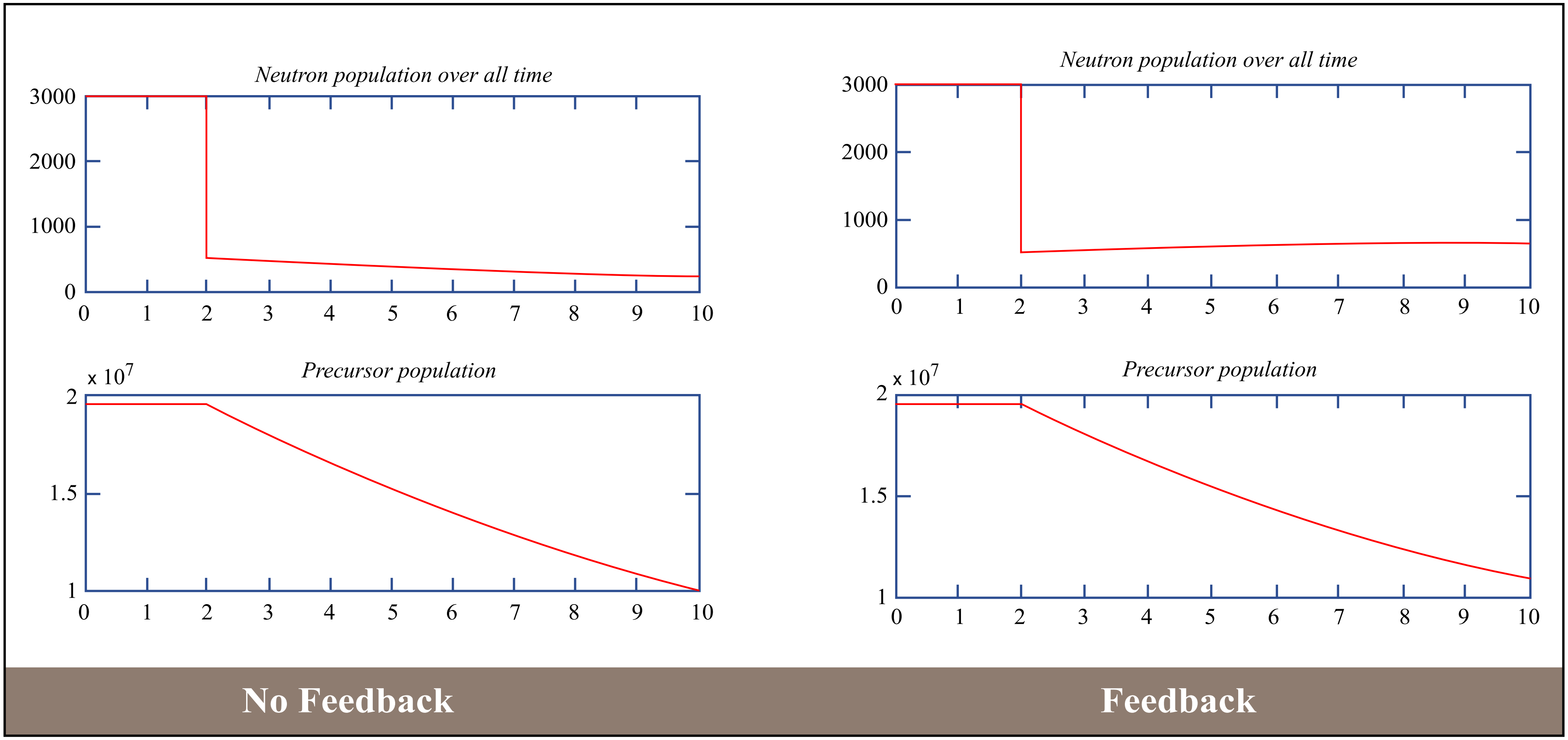
- Fuel temperature increase is greater at high power
- At low power, negative reactivity feedback is too slow, thus reactor reaches prompt critical, until temperature increases
- Both situation converge to the same power eventually

Shutdown (-5*Beta)



22.05 Neutron Science and Reactor Physics
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