Coherent and Incoherent Spin Scattering Radius

Let assume an operator for the coherent and incoherent spin as

$$\tilde{a} = \tilde{A} + \tilde{B}\vec{I}.\vec{i}$$

$$\vec{J} = \vec{I} + \vec{i}$$

$$\vec{J}^2 = \vec{I}^2 + \vec{i}^2 + 2\vec{I}.\vec{i}$$

$$\vec{I}.\vec{i} = \frac{1}{2}\left[\vec{J}^2 - \vec{I}^2 - \vec{i}^2\right]$$

$$\tilde{a}|\zeta\rangle = \tilde{A}|\zeta\rangle + \tilde{B}\vec{I}.\vec{i}|\zeta\rangle$$

$$|\zeta\rangle \rightarrow spin\ state\ eigenvector$$

$$\tilde{a}|\zeta\rangle = \tilde{A}|\zeta\rangle + \tilde{B}\frac{1}{2}\left[\vec{J}^{2} - \vec{I}^{2} - \vec{i}^{2}\right]|\zeta\rangle$$

$$a = A + B\frac{1}{2}\left[J(J+1) - I(I+1) - i(i+1)\right]$$

$$For \quad i = 1/2 \Rightarrow J = I - 1/2 \; and \; J = I + 1/2$$

$a)$ *For* $J = I - 1/2$

we have

$$a^{-} = A + \frac{1}{2}B\left[(I - 1/2)(I + 1/4) - I^{2} - I - 3/4\right]$$

or

$$a^{-} = A - \frac{1}{2}B(I+1)$$

$b)$ *For* $J = I + 1/2$

we have

$$a^{+} = A + \frac{1}{2}B\left[(I + 1/2)(I + 3/2) - I^{2} - I - 3/4\right]$$

or

$$a^{+} = A + \frac{1}{2}BI$$

A and B as a function of a^{-} *and* a^{+}

From

$$a^{+} = A + \frac{1}{2}BI$$

and

$$a^{-} = A - \frac{1}{2}B(I + 1)$$

$$A \text{ and } B \text{ become}$$

$$A = \frac{I+1}{2I+1} a^{+} + \frac{I}{2I+1} a^{-}$$

$$\text{and}$$

$$B = \frac{2}{2I+1}(a^{+} - a^{-})$$

If $a^{+} = a^{-} = a$ then $B = 0$ and $A = a$. This indicates that no spin coherent scattering exists. Hence, A relates to the coherent and B to the incoherent scattering.

The expected values for the operator $\tilde{a}^{2}$ can be derived as:

$$< a^2 >= \langle \zeta | \tilde{a} \tilde{a}^{*} | \zeta \rangle$$

$$< a^2 >= A^2 + AB < \vec{I}.\vec{i} > + BA < \vec{I}.\vec{i} > + B^2 < (\vec{I}.\vec{i})^2 >$$

The cross terms are zero since no correlation exists between the neutron spin and the nucleus spin, i.e., $< \vec{I}.\vec{i} >= 0$.

$$< a^2 >= A^2 + B^2 < (\vec{I}.\vec{i})^2 >$$

$$< (\vec{I}.\vec{i})^2 >=< (I_x i_x)^2 + (I_y i_y)^2 + (I_z i_z)^2 >$$

$$< (i_x)^2 >=< (i_y)^2 >=< (i_z)^2 >= 1/4$$

$$< (\vec{I}.\vec{i})^2 >= \frac{1}{4} < \vec{I}^2 >$$

or

$$< (\vec{I}.\vec{i})^2 >= \frac{1}{4} I(I+1)$$

$$< a^2 >= A^2 + B^2 \frac{1}{4} I(I+1)$$

With A and B as

$$A = \frac{I+1}{2I+1} a^+ + \frac{I}{2I+1} a^-$$

and

$$B = \frac{2}{2I+1} (a^+ - a^-)$$

$$< a^2 > = \left[\frac{I+1}{2I+1} a^+ + \frac{I}{2I+1} a^- \right]^2 + \frac{I(I+1)}{(2I+1)^2} (a^+ - a^-)^2$$

$$a_{coh} = \frac{I+1}{2I+1} a^+ + \frac{I}{2I+1} a^-$$

$$a_{inch} = \frac{[I(I+1)]^{1/2}}{2I+1} (a^+ - a^-)$$