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MICHAEL SHORT: So since I know series decay is a difficult topic to jump into, I wanted to quickly re-go over the derivation today and then specifically go over the case of nuclear activation analysis, which reminds me, did you guys bring in your skin flakes and food pieces? We have time. So if you didn't remember, start thinking about what you want to bring in, what you got.

AUDIENCE: Aluminum foil.

MICHAEL SHORT: OK, so you've got aluminum foil. You want to see what in it is not aluminum-- excellent. Well, what else did folks bring in?

AUDIENCE: [INAUDIBLE]

MICHAEL SHORT: OK, rubber stopper-- sound perfect. Anyone else bring something in?

AUDIENCE: [INAUDIBLE]

MICHAEL SHORT: OK, so tell you what, when you bring stuff in, bring it in a little plastic baggie. I can supply those if you don't have them with your name on them just so we know whose samples are what because that's going to be the basis for another one of your homeworks where are you going to use the stuff that we're learning today to determine which impurities and how much are in whatever thing that you looked at. And, of course, you're not going to get all the impurities because in order to do that, we'd have to do a long nuclear activation analysis, irradiate for days, and count for a longer time.

So you'll just be responsible for the isotopes on the shortlist, which we've posted on the learning module site. So again, bring in your whatever, as long as it's not hair because, apparently, that's a pain to deal with or salty because the sodium activates like crazy or fissionable, which you shouldn't have, anyway. I hope none of you have fissionable material at home.

So let's get back into series decay. We very quickly went over the definition of activity which is

just the decay constant times the amount of stuff that there is, the decay constants, and units of 1 over second. The amount of stuff-- let's call it a number density-- could be like an atoms per centimeter cubed, for example. So the activity would give you the amount of, let's say decays, per centimeter cubed per second.

If you wanted to do this for an absolute amount of a substance, like you knew how much of the substance there was, you just ditched the volume. And you end up with the activity in decays per second. That unit is better known as becquerels or BQ, named after Henri Becquerel, though I don't know if I'm saying that right. But my wife's probably going to yell at me when she sees this video.

But so becquerel is simple. It's simply 1 decay per second, and there's another unit called the curie, which is just a whole lot more decays per second. It's a more manageable unit of the case because becquerel-- the activity of many things in becquerels tends to be in the millions or billions or trillions or much, much more for something that's really radioactive. And it gets annoying writing all the zeros or all the scientific notation.

And so last time we looked at a simple situation-- let's say you have some isotope N_1 which decays with the k constant λ_1 to isotope N_2 , which decays with the k constant λ_2 and N_3 . And we decided to set up our equations in the form of change. Everyone is just a change equals a production minus a destruction for all cases.

So let's forget the activation part. For now, we're just going to assume that we have some amount of isotope, N_1 . We'll say we have $N_1(0)$ at $t = 0$. And it decays to N_2 and decays N_3 . So what are the differential equations describing the rate of change of each of these isotopes?

So how about N_1 ? Is there any method of production of isotope N_1 in this scenario? No, we just started off with some N_1 , but we do have destruction of N_1 via radioactive decay. And so the amount of changes is going to be equal to negative the activity. So for every decay of N_1 , we lose an N_1 atom. So we just put minus $\lambda_1 N_1$.

For every N_1 atom that decays, it produces an N_2 . So N_2 has an equal but different sign production term and has a similar looking destruction term. Meanwhile, since N_2 becomes N_3 , then we just have this simple term right there, and these are the differential equations which we want to solve. We knew from last time that the solution to this equation is pretty simple. I'm not going to re-go through the derivation there since I think that's kind of an easy one.

And N_3 , we know is pretty simple. We used the conservation equation to say that the total amount of all atoms in the system has to be equal to N_{10} or N_{10} . So we know we have N_{10} equals N_1 plus N_2 plus N_3 for all time. So we don't really have to solve for N_3 because we can just deal with it later.

The last thing that we need to derive is what is the solution to N_2 . And I want to correct a mistake that I made because I'm going to chalk that up to exhaustion assuming that integrating factor was zero. It's not zero, so I want to show you why it's not now.

So how do we go about solving this? What method did we use? We chose the integration factor method because it's a nice clean one. So we rewrite this equation in terms of let's just say N_2 prime-- I'm sorry-- plus $\lambda_2 N_2$ minus $\lambda_1 N_1$. And we don't necessarily want to have an N_1 in there because we want to have one variable only.

So instead of N_1 , we can substitute this whole thing in there. So $N_{10} e^{-\lambda_1 t}$ equals zero. And let's just draw a little thing around here to help visually separate.

We know how to solve this type of differential equation because we can define some integrating factor μ equals $e^{-\lambda_2 t}$. That's not too hard. I'm sorry, just $e^{-\lambda_2 t}$, not minus, of $\lambda_2 dt$. We're just equal to just $e^{-\lambda_2 t}$.

And we multiply every term in this equation by μ because we're going to make sure that the stuff here-- after we multiply by μ and μ and μ and μ for completeness-- that stuff in here should be something that looks like the end of a product rule. So if we multiply that through, we get N_2 prime $e^{-\lambda_2 t}$ plus, let's see, μ times $\lambda_2 e^{-\lambda_2 t}$ times N_2 minus $e^{-\lambda_2 t} \lambda_1 N_{10} e^{-\lambda_1 t}$ equals zero. And indeed, we've got right here what looks like the end result of the product rule where we have something, let's say, one function times the derivative of another plus the derivative of that function times the original other function.

So to compact that up, we can call that, let's say, $N_2 e^{-\lambda_2 t}$ prime minus-- and I'm going to combine these two exponents right here. So we'll have minus λ_1 and $10e^{-\lambda_2 t}$ -- let's see, is it λ_2 minus $\lambda_1 t$ equals zero. Just going to take this term to the other side of the equals sign, so I'll just do that, integrate both sides.

And we get $N_2 e^{-\lambda_2 t}$ equals-- let's see, that'll be $\lambda_1 N_{10}$ over λ_2

minus λ_1 times all that stuff. I'm going to divide each side of the equation by-- I'll use a different color for that intermediate step-- $e^{-\lambda_1 t}$. And that cancels these out. That cancels these out.

And I forgot that integrating factor again, didn't I? Yeah, so there's going to be a plus C somewhere here. And we're just going to absorb this $e^{-\lambda_1 t}$ into this integrating constant because it's an integrating constant. We haven't defined it yet.

Did someone have a question I thought I saw? OK, and so now this is where I went wrong last time because I think I was exhausted and commuted in from Columbus. I just assumed right away that C equals zero, but it's not the case.

So if we plug-in the condition at t equals 0-- and two should equal 0-- let's see what we get. That would become a zero. That t would be a zero, which means that we just end up with the equation $\lambda_1 N_1(0) - \lambda_2 N_1(0) + C = 0$ so obviously the integration constant is not like we thought it was.

So then C equals negative that stuff. That make more sense. So you guys see why the integrating constants not zero.

So in the end-- I'm going to skip ahead a little of the math because I want to get into nuclear activation analysis-- we end up with N_2 should equal, let's see, $\lambda_1 N_1(0) - \lambda_1 N_1(t) + e^{-\lambda_1 t} (N_2(0) - \lambda_1 N_1(0))$. And so since we know N_1 , we've found N_2 . We know N_3 from this conservation equation. We've now fully determined what is the concentration of every isotope in this system for all time.

And because the solution to this is not that intuitive-- like I can't picture what the function looks like in my head. I don't know about you guys. Anyone? No? OK, I can't either. I coded them up in this handy graphing calculator where you can play around with the eye of the concentration $N_1(t)$, which is just a multiplier for everything, and the relative half lives λ_1 and λ_2 .

And I'll share this with you guys, so you can actually see generally how this works. So let's start looking at a couple of cases-- move this a little over so we can see the axes. Let's say don't worry about anything before T equals zero. That's kind of an invalid part of the solution. So I'll just shrink us over there.

And so I've coded up all three of these equations. There is the solution to N_1 highlighted right

there. That's as you'd expect simple exponential decay. All N_1 knows is that it's decaying according to its own half life or exponential decay equation. And two, here in the blue, which expands, of course, looks a little more complicated.

So what we notice here is that N_2 is tied directly to the slope of N_1 . That should follow pretty intuitively from the differential equations because if you look at the slope of N_2 , well, it depends directly on the value of N_1 . For very, very short times, this is the sort of limiting behavior in the and the graphical guidance. I want to give you two solve questions like, what's on the exam or how to do a nuclear activation analysis. Is everyone comfortable with me hiding this board right here? OK.

So let's say at time is approximately zero, we know that there's going to be N_1 is going to equal about N_{10} . What's the value of N_2 going to be very, very close to 2 equals 0?

AUDIENCE: Zero.

MICHAEL SHORT: Zero. So N_2 is going to equal 0. But what's the slope of N_2 going to be? This is how we can get started solving these graphically without even knowing what the real forms are.

So we've already said that at a very short time, N_2 is approximately 0. So if that's zero, then that whole term is zero, which means that the slope of N_2 is approximately $\lambda_1 N_1$, just the activity of N_1 . And hopefully, that follows intuitively because it says for really short times before you get any buildup, the slope of N_1 determines the value of N_2 .

So if we were to start graphing these-- let's just start looking at some limiting behavior-- that's t , and we're going to need some colors for this. Let's stick with the ones on the board. Oh, hey, awesome.

Make N_1 red, N_2 blue, and N_3 green. So let's start drawing some limiting behavior. So we know that N_1 starts here at N_{10} . And we know it's going to start decaying exponentially.

So the slope here is just going to be minus λ_1 and N_1 , which is going to be the negative slope of N_2 -- looks pretty similar, doesn't it? So we know N_2 for very short times is going to start growing at the same rate that N_1 is shrinking. So we already know what sort of direction these curves are starting to go in.

How about N_3 ? What's the value of N_3 for very short times? Anyone call it out. Well, we've got kind of a solution right here. If we know that N_2 is about zero for very short times, what would

the value of n' three have to be? Also zero.

And what about the slope of N_3 ? Also, about zero. If there's no N_2 built up, then there's nothing to create N_3 .

So we know that our end three curve is going to start out pretty flat. Now how do we find some other limiting behavior? Let's now take the case-- let's see, I want to rewrite that a little closer here, so we have some room. So that's at t equals about zero. And at t equals infinity, what sort of limiting behavior do you think we'll have? What's the value of N_1 going to be at infinite time? Zero-- it will have all decayed away, will equal zero at t equals infinity. How about N_2 ?

AUDIENCE: Zero.

MICHAEL SHORT: Zero. How about N_3 ?

AUDIENCE: [INAUDIBLE]

MICHAEL SHORT: N_1 -- correct. Because of that conservation equation right here. So we know for limiting cases, N_1 one is going to be 0. N_2 is going to be 0. And N_3 , it's going to be 0.

So we've now filled in all the four corners of the graph just intuitively without solving the differential equations. Now let's start to fill in some middle parts. What other sorts of things can we determine, like, for example, where H_2 has a maximum? That shouldn't be too hard. So let's make another separation here.

So what if we want to find out when does the N_2 dt equal 0? What do we do there? Anyone have an idea? Using the equations we have up here.

AUDIENCE: [INAUDIBLE]

MICHAEL SHORT: Yeah, well, we can just take this equation right here. We can figure that out in terms of N_1 . So if D and $2D$ equals zero, then we know that $\lambda_1 N_1$ is going to equal $\lambda_2 N_2$. What this says intuitively is that the rate of production of N_2 by decaying on 1 equals the rate of destruction of N_2 by its own decay. So at some point, the N_2 is going to have to level off.

When that point is depends on the relative differences between those half lives. So we already know if we were to just kind of fill in smoothly what's going to happen, N_2 is probably going to follow something roughly looking like this.

We already know the solution to N_1 . I think we can figure that out graphically. It's simply exponential decay. The only trick now is how does N_3 shape up? What do you guys think? How would we go about graphically plotting these solutions without solving them?

I don't think yet I've given you the full form. It's kind of ugly, and I doubt that if you looked at it you'd be able to tell me exactly what it would do. So this is just the mathematical expression of N_{10} minus N_1 minus N_2 . So how do we figure out all the stuff about N_3 ? Yeah.

AUDIENCE: You could just draw a curve so that you get all three curves and always add it to the same number.

MICHAEL SHORT: Yeah.

AUDIENCE: [INAUDIBLE]

MICHAEL SHORT: Absolutely, that's totally correct. Yeah, if you just take N_{10} minus N_1 and N_2 , that gives you the value of N_3 . That's completely correct.

So you could do that sort of one point at a time and say, well, maybe around there, maybe around there. It might take a little while, though. So I want to think what's another intuitive way?

What would the value or the slope of N_3 track-- what other variable in the system? Or in other words, how are they directly related? Yeah.

AUDIENCE: The slope of n_2 are they equal?

MICHAEL SHORT: Yeah. The slope of n_3 depends directly on the value of n_2 and nothing else. So initially, you can see the value of n_2 is almost 0, so the slope of n_3 is almost 0. As the value of n_2 picks up, so should the slope of n_3 until we reach here. What happens at that point?

AUDIENCE: N_2 decreases.

MICHAEL SHORT: Yep. The rate of production of n_2 decreases because the val-- I'm sorry-- yeah, the rate of production of n_3 decreases because the rate of production of n_3 is just dependent directly on the value of n_2 . So the maximum slope of n_3 has to be right there at which point it has to start leveling off and eventually reaching 0.

You're going to see this kind of problem on the homework. You're going to see this kind of

problem on the exam. I guarantee you. But it's not going to have this exact form. But what I'll want you to be able to do is follow this example.

Let's say I pose you a small set of these first order differential equations. Can you use any method that you want-- intuitive, graphical, mathematical-- to predict what the values and slopes of these isotopes are going to be as a function of time?

So in order to get nuclear activation analysis right you need to be able to do this. In nuclear activation analysis it's just one twist. I'm going to move this over to add the twist. You're also producing isotope n_1 with a reaction rate. By some either isotope and not what you put in the reactor.

So if you want to know what your impurities and naught were to undergo what's called nuclear activation analysis, then you can figure out, depending on which one you count, what they could be.

So this right here. Let's look at the units of this versus the units of this. First of all, if we're adding them together they'd better be in the same units, right? So we already talked about the units of this decay equation. It's like number of decays per second. So this reaction right here better give us a number of atoms produced per second, or we're kind of messed up in the units. So anyone remember what is the units of-- I'll make a little extra piece right here-- what's the units of microscopic cross sections or barns? What is that in some sort of SI unit?

AUDIENCE: Centimeter squared.

MICHAEL SHORT: Yep. It's like a centimeter squared. And what about flux? This one you may not know, but it definitely depends on the number of neutrons or the number of particles that are there.

AUDIENCE: Is it barns [INAUDIBLE]

MICHAEL SHORT: Almost. So the flux describes how many particles pass through a surface in a given time. So we have how many particles per unit surface, per unit time. Ends up being neutrons per centimeter squared per second. Just like the flux of photons through a space or the flux of any particle through anywhere, it describes how many particles go through a space in a certain time.

And then there's the number of particles that are there. If we're going with atoms, it's just atoms. These are all multiplied together. The centimeter squared cancel. And we end up with

some sort of a atoms per second produced.

We can put in a little hidden unit in the cross section. If there is a reaction going on where in goes a neutron and out goes an atom or something, that should cancel all things out. Let's not get into that now.

The whole point is we have the same sort of unit going on here, which is some number of atoms produced per second. Same thing as number of atoms decayed per second. So it's the production-destruction equivalent of each other.

So, in that way, we can have a reaction rate that we impose, something artificial, by sticking something in the reactor and controlling its power level. And then follow the decay process which is a natural radioactivity event. And this is one of the simplest governing equations for nuclear activation analysis.

Now, one, I might give this to you on an exam and say OK now draw the curves for nuclear activation analysis. And maybe calculate what's the impurity level if you measure this many counts of something. Then you just work backward through the math. But I want to get you guys thinking conceptually right now. What are the real equations for nuclear activation analysis?

Let's just do these in terms of n_1 , n_2 , n_3 . That's dt , d , and $3dt$. We'll start with the stuff that's up there minus $\lambda_1 n_1$. Plus some cross section times the flux times some other atom n_0 . What other things are we missing? Are there any other methods of production or destruction of isotope n_1 that we need to consider?

Well, we've got isotope n_1 in a reactor. It can decay, or it can absorb one of the neutrons nearby. So how do we write that term-- that destruction term? Yep.

AUDIENCE: Flux times the absorption cross section
times n_0 .

MICHAEL SHORT: Yeah. So let's say that's the absorption of atom 1 times n what did you say?

AUDIENCE: Naught.

MICHAEL SHORT: And would it be n_0 or would it be n_1 ? If you want to know how quick is n_1 being destroyed--

AUDIENCE: OK.

MICHAEL SHORT: --By absorbing neutrons. So then let's call this absorption of n_0 . And is it a plus or a minus? If it's a destruction rate.

AUDIENCE: [INAUDIBLE]

MICHAEL SHORT: It's a minus. Yep. So what's really going on here is you've got some precursor isotope, whatever impurity you want to measure n_0 , producing n_1 . And you're looking at n_1 's decay signature, like its activity, to determine how much was there. But you also have to account for the fact that isotope n_1 can be burned in the reactor. So this is like producing. This is decaying. And this is we'll call it being burned.

This isn't burned in the sense of creating of fuel-- creating energy by burning fuel-- but we will refer to this sort of in-- colloquially too burning-- because we're then absorbing neutrons by n_1 and removing that from the available decay signature.

How about n_2 ? How do we modify our equation to account correctly for the production and destruction of n_2 ? And by the way this is not in the book, so I don't expect you to know it off the top of your head.

AUDIENCE: It's the same type of thing, flux.

MICHAEL SHORT: Yep So let's first take every term that we have up there. We have $\lambda_1 n_1$ minus $\lambda_2 n_2$. And what else do we have to account for? Yep.

AUDIENCE: N_2 also being burnt.

MICHAEL SHORT: That's right. So n_2 is also being burned so we'll have a minus, a flux times the absorption cross section for n_2 times the amount of n_2 .

How about n_3 ? We'll start with what we had there. $\lambda_2 n_2$. And, just like before, we've got to account for the burning of n_3 . So then we'll have minus flux times the absorption cross section of 3 times n_3 .

And these equations hold true only for the time that your material is in the reactor. What happens when you take the material out of the reactor?

AUDIENCE: You go right back to zero readings.

MICHAEL SHORT: You do. Yep. When you come out of the reactor all of the fluxes go to zero. And that's the end of that. Yeah.

AUDIENCE: Why don't you account for the production of n_2 and n_3 in the burn rate?

MICHAEL SHORT: Ah, so did I necessarily specify-- the question was why don't we account for the production of n_2 and n_3 by the burning. Right?

AUDIENCE: Yeah.

MICHAEL SHORT: Did we specify that absorbing a neutron is the way to make n_2 ?

AUDIENCE: No.

MICHAEL SHORT: Oftentimes it's not. So if you burn n_1 by absorbing a neutron, then you will make another isotope that has the same proton number and one more neutron. And it may decay by some other crazy way, or it may be stable. Who knows.

But by decay-- this could be by beta, positron, alpha, spontaneous fission-- not gamma because then you wouldn't have a different isotope. But oftentimes you won't have-- the burning process won't produce the same isotopes as the decay.

So the situation we looked at on Friday when we said let's escalate things. That was a purely hypothetical situation where isotope n_2 could be burned to make n_0 . I'm not saying it can't happen, but it's not likely.

But still we can model it. We can model anything. That just wasn't a realistic situation. This is. This is what you guys are going to have to look at to understand how much impurities there are in each of your materials.

So this I would say is the complete description of nuclear activation analysis in the reactor. At which point you then have to account for what happens when you turn the reactor off. So what actually? What physically happens when you turn the reactor off? Yep. Oh Yeah. You've answered a lot. So Chris, yeah.

CHRIS: Well, you try to-- you put your control rods all the way in and try to stop as many neutrons as you can to stop the chain reaction.

MICHAEL SHORT: Yep. So normally to shut down the reactor you'd put the control rods in and shut down the

reactor. Or the easier thing is just pull the rabbit out. Remember those little polyethylene tubes I showed you? This way we can keep the reactor on and remove your samples without changing anything.

So it makes the reactor folks-- angry would be an understatement-- to constantly change the power level of the reactor. Reactors, especially power reactors and research reactors, they're kind of like Mack trucks. If they're moving they want to stay moving, and if they're not moving they don't want to be moving. And it takes an awful lot of effort to change that.

It also happens to screw up experiments. If you are irradiating something like I was a couple months ago for 30 days, you want to have a constant flux so that your calculations are easy. You don't want 15 students to come in and turn the knobs all up and down, and then you have to account for that in your data. Which has happened.

So you guys are going to be manipulating the reactor power when the experiments are out, and it's at low power. So you're won't be infuriating anyone else on campus like we did last year. So if you didn't account for that, but they still let us in. So they're bad. Whatever.

[LAUGHING]

Yeah.

So after you either shut down the reactor or pull the rabbit out of the reactor then the production and destruction by neutrons is over but the decay keeps going. Which means if you wait too long, like for some of those short isotopes, if you wait more than a day or so, you'll have so little activity left that you won't be able to measure it.

So what we're going to be doing is sticking your samples into the reactor for maybe an hour or so, pulling them out, and immediately running them over to the detector, so that we get the most signal per unit time. Because the things are going to be the hottest when they come right out of the reactor, and every second you lose from there you lose signal. Which means you have to account for longer to get the same amount of information with the same certainty.

This is a nice segue way to what we'll be talking about Thursday which is statistics, certainty, and precision. How long do you have to count something to be confident within some interval that you've got the correct activity?

For background counts-- who here is made a NSC Geiger counter? Hopefully almost all of

you. Maybe you guys remember how long you had to count to be 95% sure that your background rate was accurate. It ends up being about 67 minutes or over an hour, and the reason is because the count rate is very low. So I'll do a little flash forward to Thursday since we're talking about it.

When you count something with a very low count rate, you have to account for longer to be as confident that your number is correct. So let's say you want to be 95% confident or within plus or minus 2 standard deviations or 2 sigma. You have to count for longer and longer.

For something that's really radioactive you can be sure, or 95% sure, that the count rate you measured is accurate for a shorter counting time. So everything in this class seems to come up in trade-offs. Right?

You trade off stability for a half-life. You trade off decay constant for half-life. You trade off binding energy for excess mass. You trade off counting time and precision. You trade off exposure and dose. which you're going to get into later.

We'll see if anyone wants to use a cell phone or eat irradiated food afterwards. And I do all the time, so that should tell you the answer.

So in the last seven minutes or so, I want to walk you through playing around with what happens when you change the values of λ_1 and λ_2 ? So what do they look like when the half-lives are roughly equal and when one is much larger than the other one?

So let's set them to be about equal. These are just unitless. So let's set them equal to one. I think the system explodes when we set them exactly equal because that term right there.

So let's say that's 1.001. It's about as close as we can get, and let's confirm that we get the same sort of behavior. So isotope n_1 just follows exponential decay. There's nothing that changes that. Isotope 2, its slope tracks the value of isotope 1 for a little while until you build up enough n_2 that it starts to decay.

You can find when that point is when $\lambda_1 n_1$ equals $\lambda_2 n_2$. There's one little step that we didn't fill in if you want to find the value of n_2 . So then you can just rearrange this a little bit, and you'll say n_2 would have to equal λ_1 over λ_2 times n_1 . Which is $n_1 e^{-\lambda_1 t}$. So if you want to find that point right there in time, you can solve this.

Then let's look at n_3 . So n_3 when n_2 is almost 0, n_3 slope is almost 0. It's a little hard to see because-- I'll tell you what-- let's make all the half lives longer which kind of expands the graph. Wrong way.

Let's make it-- Ah, we'll just move that decimal point 0.1. There we go. That's like expanding the graph. Right? So when n_2 is almost 0, the slope of n_3 is almost 0. And when n_2 reaches a maximum, so does the slope of n_3 . Just like we predicted using our graphical method right here.

And then over longer times-- let's put the half lives back to the way they were. Over long times n_3 trends to n_1 0. Don't let that little piece fool you. Again t equals less than 0 is not a valid time for this, so we're not accounting for that. And n_3 tracks right to here to the value of n_1 0, and n_2 and n_3 turn to 0. So for this case where you have the half-lives roughly equal to each other, you can expect a pretty big bump in n_2 .

What's going to happen when λ_1 is extraordinarily big meaning the half-life of n_1 is extraordinarily short. What do you guys think will happen? Not mathematically but physically. If n_1 just kind of goes ba-boom and instantly decays away.

AUDIENCE: There would be a lot of n_2 --
right then.

MICHAEL SHORT: Yep.

AUDIENCE: [INAUDIBLE]

MICHAEL SHORT: Your n_1 is just going to turn into n_2 right away, and n_2 is going to take its sweet time decaying to n_3 . So let's see what that looks like.

If λ_2 is much bigger than λ_1 , let's make the maxima a little different. Change our slider value a bit. So if λ_2 is big and λ_1 is small, well, let me change the actual axes to make this a little easier to see. There we go.

You can see that much, much more quickly than we have it in this graph right here n_1 just decays away right away. L-- I'm sorry-- n_1 decays right away. n_2 builds up to a much higher relative value because it's produced faster than it's destroyed for short amounts of time. So you can end up with a great spike in n_2 which slowly decays away to n_3 .

How about the opposite effect? What if λ_1 is really, really small indicating a very long half-life, and λ_2 is really, really large indicating a small half-life. Yeah.

AUDIENCE: It would basically go from n_1 to n_3 . As soon as it goes to n_2 , it's going to decay to n_3 .

MICHAEL SHORT: That's right. In this case you've got n_2 as soon as it's created self-destructs. So let's see what that looks like. So we can just slide λ_2 to be big, slide λ_1 to be small, and you can actually graphically see n_2 just shrink towards the x-axis.

And it's almost like you only have two equations. It's like you just have n_1 and n_3 and n_2 basically doesn't exist. Where the slope of-- where the slope of n_3 , except at extremely short times, just tracks the value of n_1 .

And I know in the book they're called secular or transient equilibria. I'm not going to require that you memorize those terms. It's more important to me that I can give you a real physical situation.

Say here's these three isotopes, for four isotopes, or six doesn't matter because we can solve these pretty quickly. Tell me what's going to happen based on the relative half-lives as long as they decay in a nice linear chain. I'm not going to give you something where n_1 can beget n_1 or n_4 or n_6 . Because at that point you can construct the equations, but I don't expect you to be able to graphically solve them.

And I may also throw you curveballs like nuclear activation analysis to see well what happens when you turn on or turn off a reactor. I've got an example of that too.

We're right at this point here, I guess, t equals 50. Yeah. I've set it up such that you turn off the reactor and n_3 is stable right there, but n_1 and n_2 continue to decay. So it's not hard to code these-- to code these sorts of things up. I'll share the links with these equations so you guys can play with them yourselves. Add to them yourselves, and try just getting an intuitive feel for how series radioactive decay happens.

So I want to know now that we spent a couple of days on it, would you guys be comfortable setting up sets of differential equations like this? Say yes, no, maybe? I see a lot of up and down shaking heads. That's a promising sign.

If not I'm willing to spend a little more time on it on Thursday if folks would like a bit of review. And if you're afraid to tell me, just send me an email anonymous or not. Yeah?

AUDIENCE: Do you think maybe Thursday we could do a like a real example?

MICHAEL SHORT: Yeah.

AUDIENCE: Of a like a series.

MICHAEL SHORT: I think so. Yeah, with real example with numbers and everything.

AUDIENCE: Yeah.

MICHAEL SHORT: Sure. OK. Well, we can make one of those up for Thursday. Cool.

And what about the graphical solution method since I don't know whether they teach that in the GIRs, but what I do want to be able to do is look at the limiting cases. In other words, fill in the four corners of the graph.

At t equals 0, what are things actually doing? n_1 is just decaying at its half-life. There is no n_2 yet. So these slopes are equal and opposite. And there's no n_3 yet, so that there is no slope of n_3 .

So I would like you guys to try reproducing this. And I will-- again I'll provide pictures of these blackboards so you guys can see, but it would be very helpful for you guys to try to reproduce these graphs as we saw them.

Then you can check them here on the graphical calculator, and then play around with the amount of n_0 , or-- I think I just broke it. Let's just call that one. There we go. And play around with sliders or values of n_1 , n_2 , or n_3 . That's an interesting solution.

So since it's about four or five of, I want to open it up to any questions you guys may have. Yeah.

AUDIENCE: I have a question from the [INAUDIBLE] First time do this. Did you have-- do you know what integrated video because like there are endless possibilities if you just [INAUDIBLE] add up to the right mass number?

MICHAEL SHORT: Oh yeah. The questions for a spontaneous fission. What fission products do you choose? I'll say they're all good. As long as you pick something with roughly equalish masses, so you don't pick like it fizzes into hydrogen and something quite smaller, which should be better known as just proton emission. You're going to get roughly the same answer. Yep.

AUDIENCE: Would that be something we just like set up. Like the top number is 80. We just look at whatever 40, 42, 38 arms pick a number.

MICHAEL SHORT: Roll a D 80. You'll get basically the same result. Roll an 80 sided dice. Hopefully at MIT you could find one, or write a program to make random number between about 10 and 80 or 10 and 70.

Let's go to the actual problem set to see what you guys mean. I want to make sure I'm answering the correct question. Problem statement Yep. Yep. So allowable this is for this one allowable nuclear reactions. Yep. So for a spontaneous fission just pick one you think would be likely.

You can also look up what sort of isotopes are created when elements fission. It's not straight down the middle, so like uranium won't often split into two equally sized fission products. They'll have roughly different masses, but which ones you pick? You're still going to get the same general solution. Yep.

AUDIENCE: I'm so confused on how to find one. Like a situation where it is unlikely possible because is it spontaneous fission or is it generally possible for heavier elements like transuranic elements?

MICHAEL SHORT: So the question is when is spontaneous fission possible? Is it only for heavy elements? There is a difference between energetically possible and observed. That's part of the trick to this problem. If you do out the Q equation to find out for each fission product that you picked, you may be surprised at the result.

However, you're right. You don't tend to see spontaneous fission happen until you get to really heavy things like uranium. So there's more to will something spontaneously fission than does the Q value allow it to happen.

So I don't want to give away anymore but I will say if you're surprised at your result, you might be right. Yep.

AUDIENCE: On this question for electron capture. In the equation you gave us, you're calculating Q in an electronic capture. It's the massive parents minus the daughters minus the I think binding energy of the electrons is what you wanted. When I try find out what the binding energy of an electron is, it says it depends on the shell that it's in.

MICHAEL SHORT: Yep.

AUDIENCE: So how do we know which electron the nucleus is after? Do we assume its from the innermost shell?

MICHAEL SHORT: Yep. So the question was if you're doing electron capture where you have some parent nucleus and you've got a lot of electron shells. The binding energy of every electron is different. Which one goes in? One, you find the data on the nest tables. Two, chances are it will be the closest one. So a roughly 80% of the time these things happen from the K shell with very decreasing probabilities from the outer shells.

So you can pick either the k or the L shell, like both things may happen. But I would say for simplicity's sake assume it's an inner most shell electron. And you can look up the binding energy on the NIST tables on the learning module site. So any other questions? Yeah, Luke.

LUKE: On graphing the spectrum, the satellite intensity vs. the energy for 4 2.

MICHAEL SHORT: For 4 2. Ah yes. So graphing the-- this would be like if you had an electron detector. Is that what I asked for? 4 2, write the full nuclear reactions and draw the energy spectrum you expect from each released form of radiation including secondary ejections of particles or photons. So by a spectrum, I mean, yep, energy versus intensity.

LUKE: OK.

MICHAEL SHORT: So there you'll have to account for the spectrum like the various range of the betas that can be released, any ejected electrons, any Auger electrons, any photons from X-ray emission from electrons falling down and energy levels. Yeah, Alex, you had a question?

ALEX: Yeah. What are the Auger electrons?

MICHAEL SHORT: The Auger electrons is that funny case where in our mental model a gamma ray hits an inner shell electron, and it's usually an inner shell electron, shooting it out. Then another electron will fall down to fill that hole emitting an X-ray. And the Auger process can be thought of that second X-ray hits an electron on the way out and fires out the electron.

So this here would be the Auger electron. They tend to be particularly low energy. Yeah, Luke.

LUKE: If you have that cascade of electrons during an electron capture, are they still Auger [INAUDIBLE] radiation?

MICHAEL SHORT: Yep. As long as you have a higher level shell coming down to a lower shell and the ejection of an outer shell electron. That's than Auger electron emission process. Regardless of whether it started with gamma or started with electron capture. Yep.

AUDIENCE: How do we know if that happens?

MICHAEL SHORT: You can actually sense or detect the energy of those Auger electrons with a very sensitive Auger electron detector. These are in the sort of hundreds or thousands of EV range.

AUDIENCE: But for like the context of this question, how do we know-- where would we go to find them?

MICHAEL SHORT: Oh, to get the Auger electron data?

AUDIENCE: Yeah.

MICHAEL SHORT: For that you can actually look up the binding energies of an outer shell electron, and you can do that energy balance where it would be E_2 minus E_1 . Whatever the energy of that X-ray is minus the binding energy of the emitted electron. And because there's infinite possibilities. I mean you could eject any electron, just pick one. And say here's an Auger electron, or draw a couple of lines in places. I don't have I don't want you to get every single line. If we asked you to do this for uranium, there's like you know 92 electrons and a lot of different transitions that's not what we're going for. I want to make sure you know the physics. Not that you can draw 92 lines accurately with a fine-toothed pencil. Do you need a question two?

AUDIENCE: Yeah. So for 2 1 if we write two possible nuclear reactions for 239 on [INAUDIBLE] the right was only off the case of any decision and that it's a state which decay processes and repeating processes may be possible for each general type of reaction. What exactly does that-- I find the answer for number two is alpha.

MICHAEL SHORT: Maybe that's the answer.

AUDIENCE: Oh, OK.

MICHAEL SHORT: And does anything compete with alpha decay? Does anything compete with spontaneous fission? OK. Cool.

[INTERPOSING VOICES]

AUDIENCE: Is that an indication of [INAUDIBLE]

MICHAEL SHORT: Beta decay. Well, is there-- well, for that you can look up the table of nuclides which I've got up here. So let's take a look at plutonium 239, and it precedes by alpha or spontaneous fission. So every year I switch up the isotope and make sure that there's at least a couple of decay modes, and therefore the answers are going to change every year. But the general question doesn't,

So this year I happened to pick an interesting one. Yeah, it's kind of a mind game, right? What are you missing?

AUDIENCE: Nothing.

MICHAEL SHORT: Nothing. Yeah.

[LAUGHING]

Yeah, go with your physical intuition. Any other questions, and maybe time for one more.

AUDIENCE: For 3 2, I could only find one nuclear reaction. The action played after the nuclear reaction.

MICHAEL SHORT: Ah Yeah,

AUDIENCE: That's very curious you're really [INAUDIBLE] decay.

MICHAEL SHORT: Yeah.

AUDIENCE: And, so I was wondering where the molybdenum [INAUDIBLE]

MICHAEL SHORT: Yeah, so the question is I specifically wrote which nuclear reactions could make 99 molybdenum, despite there being only one natural one. So what could you induce artificially? And if you can do that profitably, I'll guarantee you there's a startup in it for you.

So what are all the different particles that something could absorb to create molybdenum 99, and which of those are allowable nuclear reactions? And if they're not allowable, how much energy do you have to put in an accelerator to make that reaction happen? And is the price of electricity in the accelerator worth the Molly 99 that you create?

There are actually quite a few startups working on this problem right now. So the answer to this question is be creative. Think about all the different particles you know of, and how they

could create Molly 99. And figure out are any of those processes allowed, and if they're not allowed, how energetic do you have to make the incoming particles to allow them?

Ah, good question. There's some creativity hiding in these problems. So it's 10:02. I want to cut it off here, and we'll start off Thursday with a numerical example of this stuff. Nuclear activation analysis.