8.701

Introduction to Nuclear and Particle Physics

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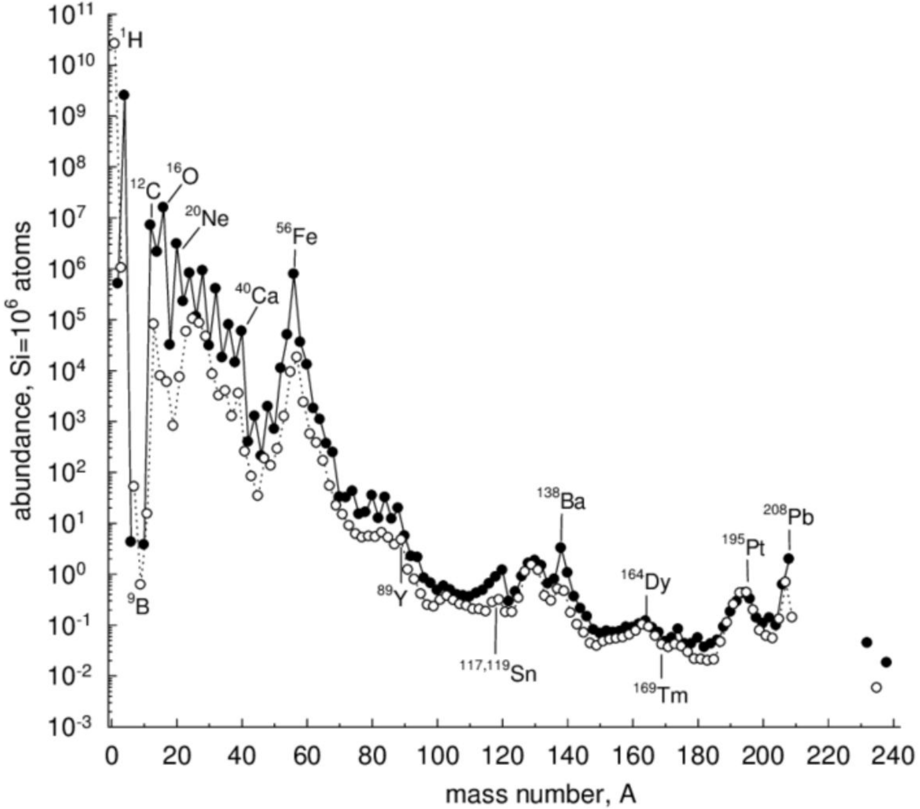
9. Nuclear Physics

9.2 Binding Energy

Nuclear Abundance in the Solar System



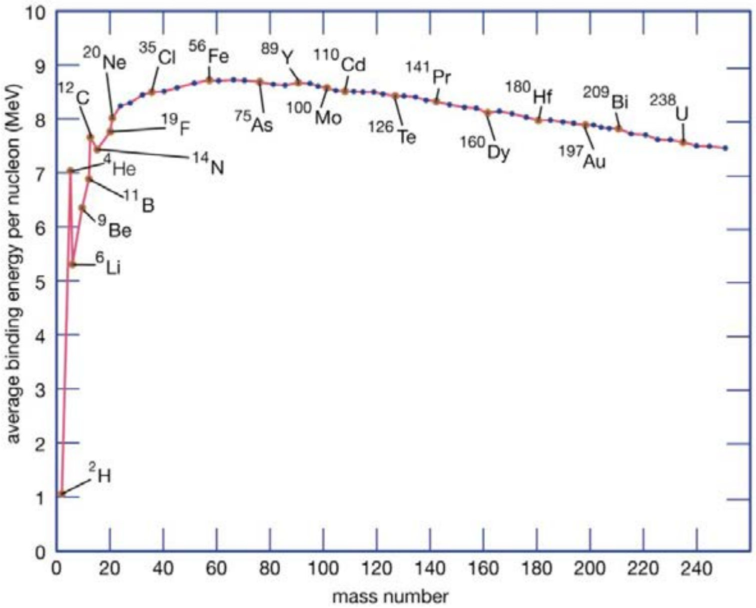
Binding Energy

$$B(Z, A) = [Z(M_p+m_e) + (A-Z)M_N - M(A,Z)]\ c^2$$

$$M_p = 938.272\ \text{MeV}/c^2 = 1836.149\ m_e$$

$$M_N = 939.566\ \text{MeV}/c^2 = 1838.679\ m_e$$

$$m_e = 0.511\ \text{MeV}/c^2$$



Apart from the lightest elements, binding energy per nucleon is about 7.5-9 MeV

Parametrization of Binding Energies

Weizäcker (semi-empirical mass) formula and liquid-drop model

$$M(A,Z) = NM_n + ZM_p + Zm_e - a_V A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_a \frac{(N-Z)^2}{4A} + \frac{\delta}{A^{1/2}}$$

with $N = A - Z$.

$a_V = 15.67$ MeV/c^2, $a_s = 17.23$ MeV/c^2, $a_c = 0.714$ MeV/c^2, $a_a = 93.15$ MeV/c^2 and $\delta = -11.2, 0, +11.2$ MeV/c^2 for even Z and Z, odd A, or odd Z and N, respectively.

Liquid-Drop Model

Volume term: $a_V A$

Dominates binding energy, proportional to the number of nucleon which contributes with about 16 MeV each.

Nuclear force must be short range, corresponding roughly to the distance between two nucleons.

As a result, the central density is ~0.17 nucleons / fm^3 and the average distance ~1.8fm.

Liquid-Drop Model

Surface term: $a_s A^{2/3}$

Nucleons at the surface of the nucleus are surrounded by fewer nucleons and the binding energy is reduced

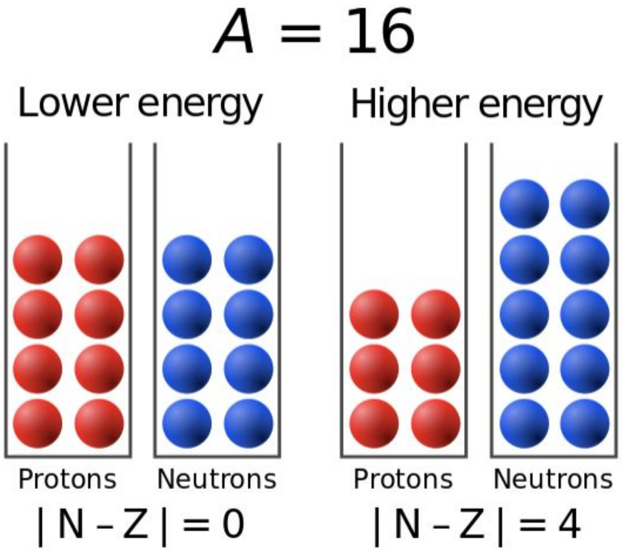
Coulomb term: $a_c \frac{Z^2}{A^{1/3}}$

Repulsive force acting between protons reduces the binding energy

Liquid-Drop Model

Asymmetry term: $a_a \frac{(N-Z)^2}{4A}$

Pauli exclusion principle allows two identical fermions to occupy the same state. Additional particles will occupy higher energy levels. Hence, an asymmetry in number of protons and neutrons reduces the binding energy.



Liquid-Drop Model

Pairing term: $\frac{\delta}{A^{1/2}}$

Again, the Pauli exclusion principle leads to a reduced binding energy when Z and N are not even

$$\delta(A,Z) = \begin{cases} -\delta_0 & Z, N \text{ even } (A \text{ even}) \\ 0 & A \text{ odd} \\ +\delta_0 & Z, N \text{ odd } (A \text{ even}) \end{cases}$$

Liquid-Drop Model