8.701

Introduction to Nuclear and Particle Physics

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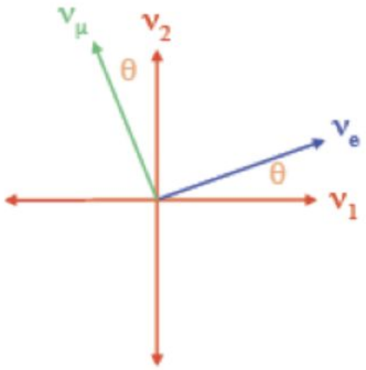
8. Neutrinos

8.3 Mixing

Neutrino Mixing



The neutrino flavor states in bra-ket notation.

For Two Neutrinos....

flavor mass

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

The mixing of the states is expressed by a rotation matrix.

$$|\nu_e\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$

$$|\nu_\mu\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$

Neutrino Mixing

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

So starting with the mixing matrix.

$$|\nu_\mu(0)\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle$$

The state at time t=0.

$$|\nu_\mu(t)\rangle = -\sin\theta e^{-iE_1 t}\,|\nu_1\rangle + \cos\theta e^{-iE_2 t}\,|\nu_2\rangle$$

The state's evolution in time.

Then the probability is given by the amplitude squared.

$$P_{osc} = |\langle\nu_e|\nu_\mu(t)\rangle|^2 = \frac{1}{2}\sin^2 2\theta(1 - \cos(E_2 - E_1)t)$$

Neutrino Mixing

$$P_{osc} = |\langle \nu_e | \nu_\mu(t) \rangle|^2 = \frac{1}{2}\sin^2 2\theta (1 - \cos(E_2 - E_1)t)$$

$$E_i = \sqrt{p^2 - m_i^2} \approx p + m_i^2/2p$$

$$t/p = L/E$$

We know the mass is small so we can use a Taylor expansion and then change some units.

$$P_{osc} = \frac{1}{2}\sin^2 2\theta \left(1 - \cos\left(\frac{(m_2^2 - m_1^2)L}{4E}\right)\right)$$

$$P_{osc} = \sin^2 2\theta \sin^2\left(\frac{1.27\Delta m^2 L}{E}\right)$$

Look! It depends on mass differences, so if neutrinos oscillate they must have mass!

Neutrino Mixing

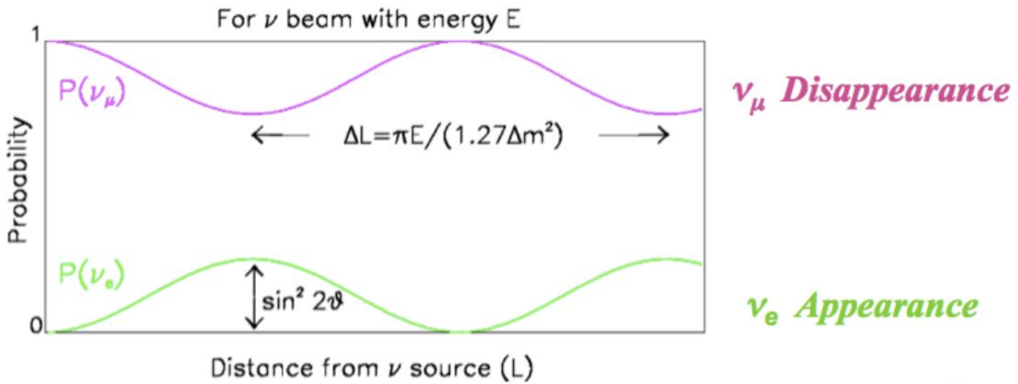
$$P_{Osc} = \sin^2 2\theta \sin^2\left(1.27\Delta m^2 L / E\right)$$

...Depends Upon Two Experimental Parameters:

- L – The distance from the ν source to detector (km)
- E – The energy of the neutrinos (GeV)

...And Two Fundamental Parameters:

- $\Delta m^2 = m_1^2 - m_2^2 \quad (eV^2)$
- $\sin^2 2\theta$

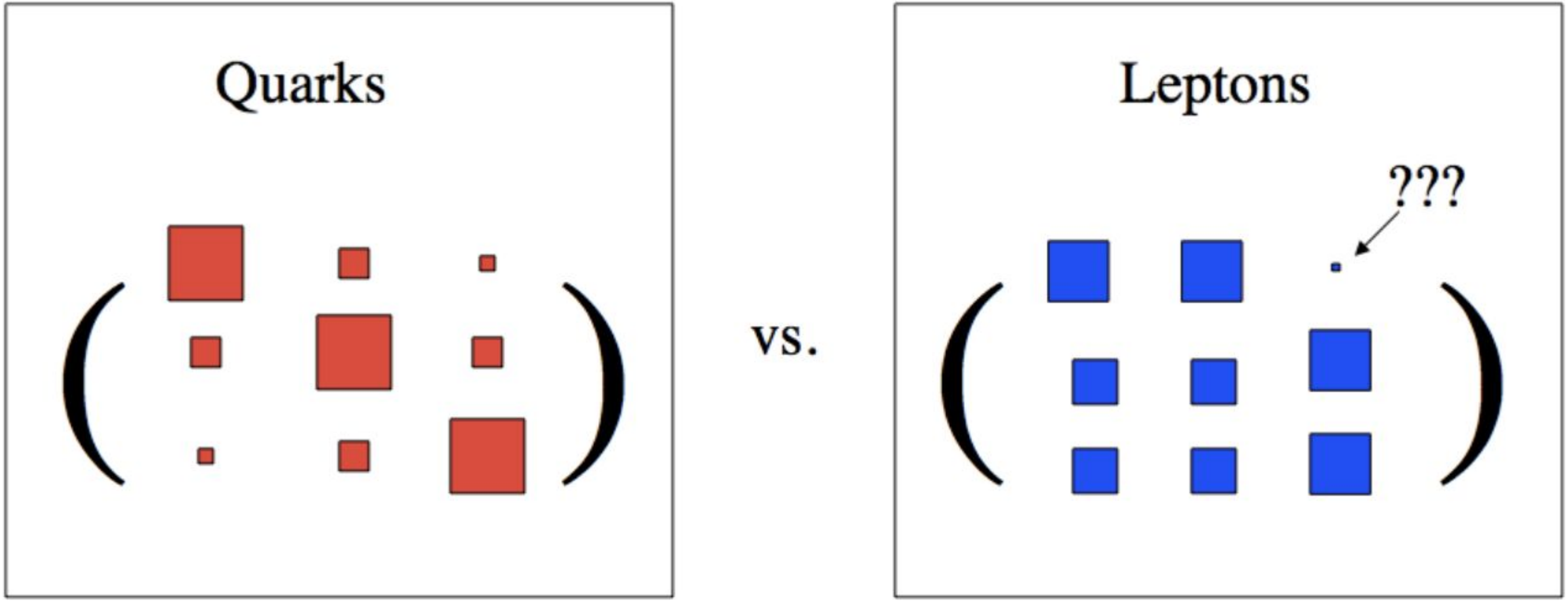


Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{\mathrm{CP}}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{\mathrm{CP}}} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} e^{i\eta_1} & 0 & 0 \\ 0 & e^{i\eta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta_{\mathrm{CP}}} \\ -s_{12}\,c_{23} - c_{12}\,s_{13}\,s_{23}\,e^{i\delta_{\mathrm{CP}}} & c_{12}\,c_{23} - s_{12}\,s_{13}\,s_{23}\,e^{i\delta_{\mathrm{CP}}} & c_{13}\,s_{23} \\ s_{12}\,s_{23} - c_{12}\,s_{13}\,c_{23}\,e^{i\delta_{\mathrm{CP}}} & -c_{12}\,s_{23} - s_{12}\,s_{13}\,c_{23}\,e^{i\delta_{\mathrm{CP}}} & c_{13}\,c_{23} \end{pmatrix}$$

Mixing



Oscillations

$$P_{\alpha\beta} = |\langle \nu_\beta | \nu_\alpha(t) \rangle|^2 = |\sum_{i=1}^{n} \sum_{j=1}^{n} U_{\alpha i}^{*} U_{\beta j} \langle \nu_j | \nu_i(t) \rangle|^2$$