8.701

Introduction to Nuclear and Particle Physics

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6. Weak Interaction

6.4 Quarks

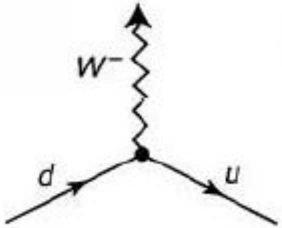
Charged Weak Interaction of Quarks

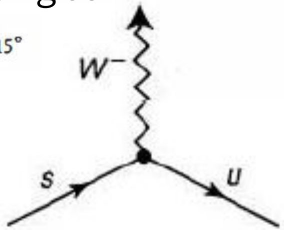
$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}, \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \qquad \text{(lepton generations)}$$

$$\begin{pmatrix} u \\ d \end{pmatrix}, \quad \begin{pmatrix} c \\ s \end{pmatrix}, \quad \begin{pmatrix} t \\ b \end{pmatrix} \qquad \text{(quark generations)}$$

Cabibbo angle

$\theta_C = 13.15°$



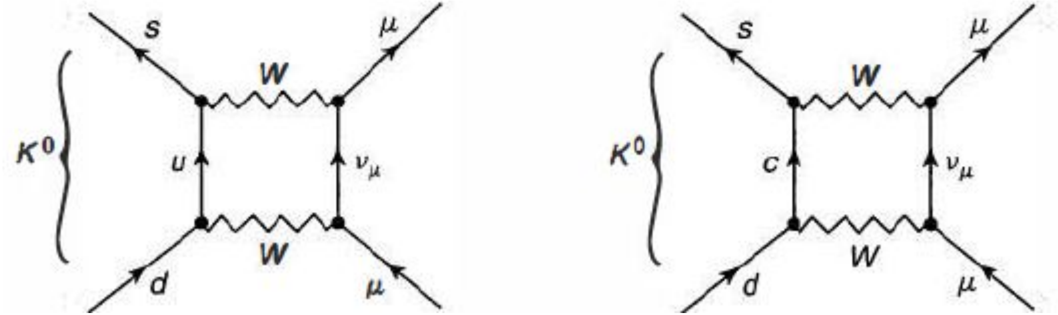


$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)\cos\theta_C \qquad \frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)\sin\theta_C$$

$$\frac{\Gamma(K^- \to l^- + \bar{\nu}_l)}{\Gamma(\pi^- \to l^- + \bar{\nu}_l)} = \tan^2\theta_C \left(\frac{m_\pi}{m_K}\right)^3 \left(\frac{m_K^2 - m_l^2}{m_\pi^2 - m_l^2}\right)^2$$

Weak interaction respects lepton generations but not quark generations

Charged Weak Interaction of Quarks



Observation: amplitude proportional to $-\sin\Theta_c * \cos\Theta_c$
indicated a 4th quark

“Correct” states to use in weak interaction are

$$d' = d\cos\theta_C + s\sin\theta_C, \quad s' = -d\sin\theta_C + s\cos\theta_C$$

Charged Weak Interaction of Quark

Matrix form

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_C + s\sin\theta_C \end{pmatrix}, \quad \begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ -d\sin\theta_C + s\cos\theta_C \end{pmatrix}$$

Kobayashi and Maskakwa generalized scheme for 3 generations to CKM matrix with 3 independent angles and one complex phase

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$\begin{bmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{bmatrix} = \begin{bmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351^{+0.00015}_{-0.00014} \\ 0.22520 \pm 0.00065 & 0.97344 \pm 0.00016 & 0.0412^{+0.0011}_{-0.0005} \\ 0.00867^{+0.00029}_{-0.00031} & 0.0404^{+0.0011}_{-0.0005} & 0.999146^{+0.000021}_{-0.000046} \end{bmatrix}$$

CKM Parametrization

"Standard" parameters

$$\begin{bmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{bmatrix}\begin{bmatrix}c_{13} & 0 & s_{13}e^{-i\delta_{13}}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta_{13}} & 0 & c_{13}\end{bmatrix}\begin{bmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}}\\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13}\\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13}\end{bmatrix}$$

$\theta_{12} = 13.04 \pm 0.05°$, $\theta_{13} = 0.201 \pm 0.011°$, $\theta_{23} = 2.38 \pm 0.06°$, and $\delta_{13} = 1.20 \pm 0.08$ radians.

Wolfenstein parameters

$$\begin{bmatrix}1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta)\\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2\\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1\end{bmatrix} + O(\lambda^4)$$

$\lambda = 0.2257^{+0.0009}_{-0.0010}$, $A = 0.814^{+0.021}_{-0.022}$, $\rho = 0.135^{+0.031}_{-0.016}$, and $\eta = 0.349^{+0.015}_{-0.017}$

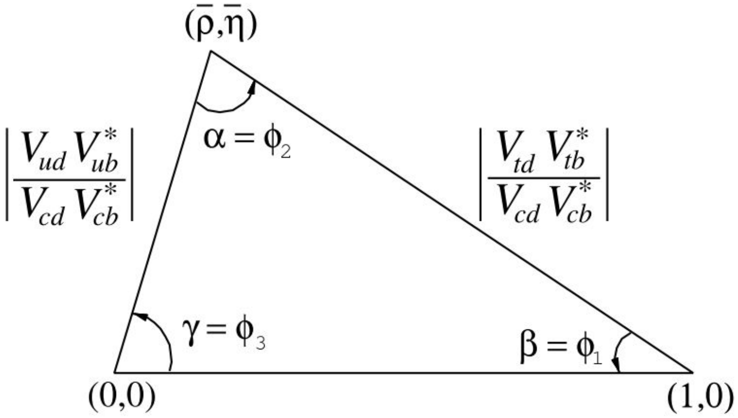
Unitarity Triangle

Unitarity puts constraints on parameter values

$\sum_i V_{ij} V_{ik}^* = \delta_{jk}$ and $\sum_j V_{ij} V_{kj}^* = \delta_{ik}$

The six vanishing combinations can be represented as triangles, like

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$



Unitarity Triangle