

8.701

Introduction to Nuclear
and Particle Physics

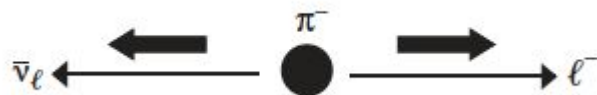
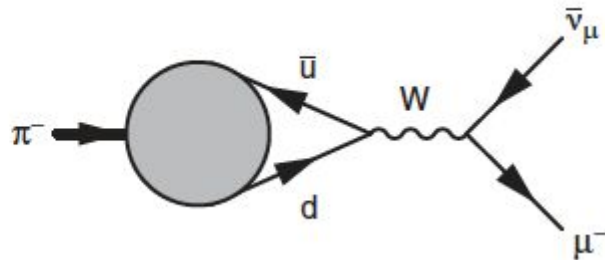
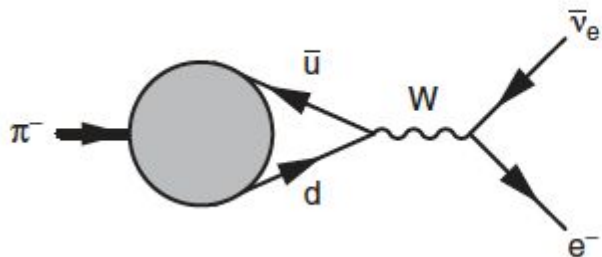
Markus Klute - MIT

6. Weak Interaction

6.3 Pion Decay



Pion Decay

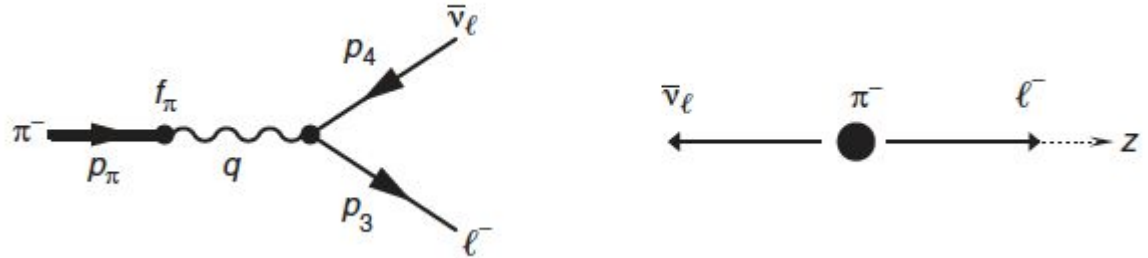


$$\frac{\Gamma(\pi^- \rightarrow e^- \bar{\nu}_e)}{\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)} = 1.230(4) \times 10^{-4}.$$

$$u_\uparrow \equiv \frac{1}{2} \left(1 + \frac{P}{E+m} \right) u_R + \frac{1}{2} \left(1 - \frac{P}{E+m} \right) u_L.$$

Pion Decay Rate

Pion rest frame



$$p_\pi = (m_\pi, 0, 0, 0), \quad p_\ell = p_3 = (E_\ell, 0, 0, p) \quad \text{and} \quad p_{\bar{\nu}} = p_4 = (p, 0, 0, -p),$$

Weak leptonic current

$$j_\ell^\nu = \frac{g_W}{\sqrt{2}} \bar{u}(p_3) \frac{1}{2} \gamma^\nu (1 - \gamma^5) v(p_4).$$

Matrix element

$$\begin{aligned} \mathcal{M}_{fi} &= \left[\frac{g_W}{\sqrt{2}} \frac{1}{2} f_\pi p_\pi^\mu \right] \times \left[\frac{g_{\mu\nu}}{m_W^2} \right] \times \left[\frac{g_W}{\sqrt{2}} \bar{u}(p_3) \gamma^\nu \frac{1}{2} (1 - \gamma^5) v(p_4) \right] \\ &= \frac{g_W^2}{4m_W^2} g_{\mu\nu} f_\pi p_\pi^\mu \bar{u}(p_3) \gamma^\nu \frac{1}{2} (1 - \gamma^5) v(p_4), \end{aligned}$$

Pion Decay Rate

$$\begin{aligned}\mathcal{M}_{fi} &= \frac{g_W^2}{4m_W^2} f_\pi m_\pi \cdot \frac{m_\pi + m_\ell}{\sqrt{2}m_\pi} \cdot \left(\frac{m_\pi^2 - m_\ell^2}{2m_\pi} \right)^{\frac{1}{2}} \cdot \frac{2m_\ell}{m_\pi + m_\ell} \\ &= \left(\frac{g_W}{2m_W} \right)^2 f_\pi m_\ell (m_\pi^2 - m_\ell^2)^{\frac{1}{2}}.\end{aligned}$$

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}.$$

$$\langle |\mathcal{M}_{fi}|^2 \rangle \equiv |\mathcal{M}_{fi}|^2 = 2G_F^2 f_\pi^2 m_\ell^2 (m_\pi^2 - m_\ell^2),$$

$$\Gamma = \frac{4\pi}{32\pi^2 m_\pi^2} \text{P} \langle |\mathcal{M}_{fi}|^2 \rangle = \frac{G_F^2}{8\pi m_\pi^3} f_\pi^2 [m_\ell (m_\pi^2 - m_\ell^2)]^2$$

$$\frac{\Gamma(\pi^- \rightarrow e^- \bar{\nu}_e)}{\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)} = \left[\frac{m_e (m_\pi^2 - m_e^2)}{m_\mu (m_\pi^2 - m_\mu^2)} \right]^2 = 1.26 \times 10^{-4},$$

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Fall 2020

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