8.701

Introduction to Nuclear and Particle Physics

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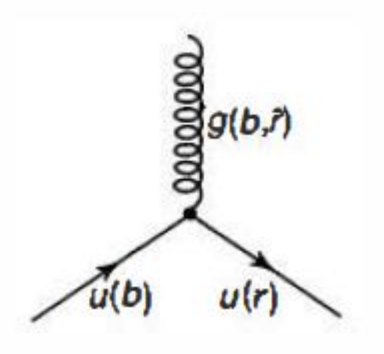
5. QCD

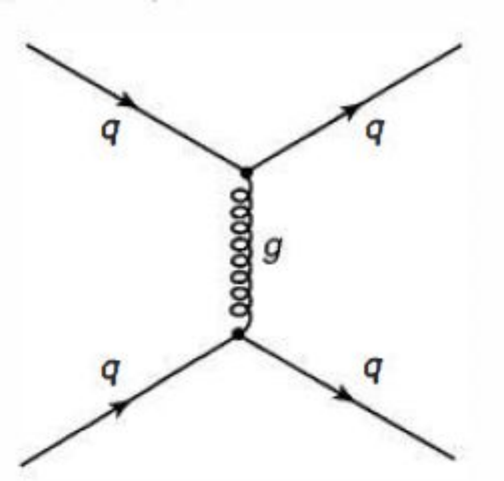
5.3 Feynman Rules for QCD

QCD

In QCC, color plays the role of charge.

The fundamental process analog to e→e+ɣ in QED is q→q+g





Three kind of charges

Quarks come in three colors: **red**, **green**, and **blue**

In addition to the spinor we need to keep track of colors with a three-element vector

$$c = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{ for red, } \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ for blue, } \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{ for green}$$

8 Gluons

QCD based on SU(3) symmetry group with 8 independent "rotations"

Keep track of the color state of the gluon we need an eight-element column vector

$$|1\rangle = (r\bar{b} + b\bar{r})/\sqrt{2} \qquad |5\rangle = -i(r\bar{g} - g\bar{r})/\sqrt{2}$$

$$|2\rangle = -i(r\bar{b} - b\bar{r})/\sqrt{2} \qquad |6\rangle = (b\bar{g} + g\bar{b})/\sqrt{2}$$

$$|3\rangle = (r\bar{r} - b\bar{b})/\sqrt{2} \qquad |7\rangle = -i(b\bar{g} - g\bar{b})/\sqrt{2}$$

$$|4\rangle = (r\bar{g} + g\bar{r})/\sqrt{2} \qquad |8\rangle = (r\bar{r} + b\bar{b} - 2g\bar{g})/\sqrt{6}$$

$$a = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ for } |1\rangle,$$

Notation

SU(3) “Pauli matrices”

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \qquad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Commutators:

$$[\lambda^\alpha, \lambda^\beta] = 2i f^{\alpha\beta\gamma} \lambda^\gamma \qquad f^{123} = 1, \quad f^{147} = f^{246} = f^{257} = f^{345} = f^{516} = f^{637} = \frac{1}{2},$$

$$f^{458} = f^{678} = \sqrt{3}/2$$

Feynman Rules for QCD

External Lines:

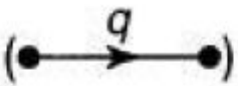
$$\text{Quark}: \begin{cases} \text{Incoming} (\rightarrow\bullet) : u^{(s)}(p)c \\ \text{Outgoing} (\bullet\rightarrow) : \bar{u}^{(s)}(p)c^{\dagger} \end{cases}$$

$$\text{Antiquark}: \begin{cases} \text{Incoming} (\leftarrow\bullet) : \bar{v}^{(s)}(p)c^{\dagger} \\ \text{Outgoing} (\bullet\leftarrow) : v^{(s)}(p)c \end{cases}$$

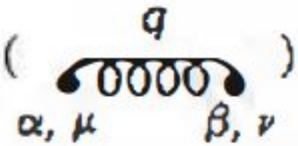
$$\text{Gluon}: \begin{cases} \text{Incoming} (\xrightarrow{\alpha,\,\mu} \bullet) : \epsilon_{\mu}(p)a^{\alpha} \\ \text{Outgoing} (\bullet \xleftarrow{\alpha,\,\mu}) : \epsilon^{*}_{\mu}(p)a^{\alpha *} \end{cases}$$

Feynman Rules for QCD

Propagator:

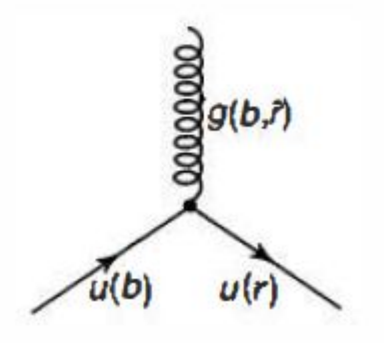
Quarks and antiquarks: (

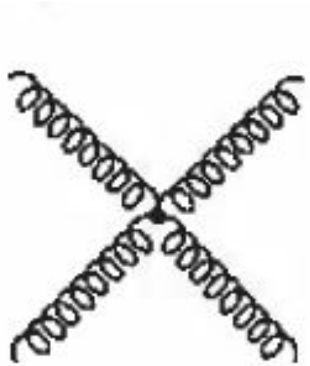
) : $\dfrac{i(\not{q} + mc)}{q^2 - m^2c^2}$

Gluons: (

) : $\dfrac{-ig_{\mu\nu}\delta^{\alpha\beta}}{q^2}$

Fundamental vertices

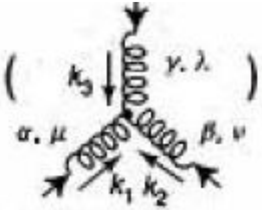




Feynman Rules for QCD

Vertices:

Quark-gluon: (): $\frac{-ig_s}{2}\lambda^\alpha\gamma^\mu$

Three gluon: ():

$$-g_s f^{\alpha\beta\gamma}[g_{\mu\nu}(k_1 - k_2)_\lambda + g_{\nu\lambda}(k_2 - k_3)_\mu + g_{\lambda\mu}(k_3 - k_1)_\nu]$$

Four gluon: () :

$$-ig_s^2[f^{\alpha\beta\eta}f^{\gamma\delta\eta}(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}) + f^{\alpha\delta\eta}f^{\beta\gamma\eta}(g_{\mu\nu}g_{\lambda\rho} - g_{\mu\lambda}g_{\nu\rho})$$
$$+f^{\alpha\gamma\eta}f^{\delta\beta\eta}(g_{\mu\rho}g_{\nu\lambda} - g_{\mu\nu}g_{\lambda\rho})]$$