8.701

Introduction to Nuclear and Particle Physics

Markus Klute - MIT

4. QED

4.4 Photons

Quantum Electrodynamic

Relativistic quantum field theory of electrodynamics describing how light and matter interacts.

QED describes all phenomena involving electrically charged particles interacting by **photon** exchange

Photon is an elementary particle, the quantum of the electromagnetic field

QED can be described as a perturbation theory and provides extremely accurate predictions. It’s “our pride and joy!”

Classical Electrodynamic: Maxwell's equation

$$\vec{\nabla} \cdot \vec{E} = \rho \qquad \text{(Gauss)},$$

$$\vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{j} \qquad \text{(Ampère)},$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \text{(Gauss)},$$

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0 \qquad \text{(Faraday)}.$$

$$\vec{B} = \vec{\nabla} \times \vec{A}, \qquad \vec{E} = -\vec{\nabla}\Phi - \frac{\partial \vec{A}}{\partial t},$$

Maxwell's equation

$$A^\mu = (\Phi, \vec{A}) \quad \text{and} \quad j^\mu = (\rho, \vec{j})$$

$$\Box \equiv \partial^\mu \partial_\mu = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2$$

$$\Box A^\mu - \partial^\mu(\partial_\nu A^\nu) = j^\mu$$

$$\partial_\mu F^{\mu\nu} = j^\nu, \quad \text{with} \quad F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu.$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

Gauge

$$\Box A^{\mu} - \partial^{\mu}(\partial_{\nu}A^{\nu}) = j^{\mu}$$

$$A^{\mu} \rightarrow A'^{\mu} = A^{\mu} + \partial^{\mu}\chi\,;$$

Coulomb gauge $\Box A^{\mu} = j^{\mu}$

QED

$\mathbf{A^\mu}$ becomes the wave function of the photon satisfying $\Box A^\mu = 0$

$$A^\mu(x) = a e^{-(i/\hbar)p\cdot x}\epsilon^\mu(p)$$

$\boldsymbol{\varepsilon^\mu}$ is the polarization vector and **a** a normalization factor.

We find

$$p^\mu p_\mu = 0, \quad \text{or} \quad E = |\mathbf{p}|c$$

Polarization vector

The choice $A^0 = 0$ and $\nabla \cdot \mathbf{A} = 0$ of gauge requires that

$$\epsilon^0 = 0, \qquad \boldsymbol{\epsilon} \cdot \mathbf{p} = 0$$

i.e. the three-vector is perpendicular to the direction of propagation -> a free photon is transversely polarized.

Photons have two independent solutions (polarization states) for a given momentum.