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Lecture Notes for 8.225 / STS.042, "Physics in the 20th Century": Energy from Nuclear Fission

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Introduction

These notes discuss the earliest estimates for the energy released each time a heavy, unstable nucleus undergoes nuclear fission. The goal is to follow arguments like those first pieced together by physicists Lise Meitner and Otto Robert Frisch in December 1938, using approximate order-of-magnitude estimates. Those rough estimates were sufficient to convince scientists all around the world — and, before long, politicians and military officials — that the energies involved in nuclear reactions like fission were enormously greater than those associated with familiar chemical reactions. In particular, rough-and-ready estimates like these helped to convince leaders in multiple countries to pursue nuclear weapons programs, right on the eve of the outbreak of fighting in the Second World War.

The first section gives a brief review of the kinds of nuclear transmutations that had already become familiar to nuclear physicists and chemists by the late 1930s. In the second section, we turn to estimates of the energy released from nuclear fission.

Reading these notes is *optional*; the notes are meant to fill in some of the gaps in various derivations that we will not cover during our class session.

Nuclear Transmutations

Beginning in the mid-1890s, led by pioneering efforts by Marie and Pierre Curie, physicists and chemists studied and classified several types of radioactive transformations among chemical elements. Following Ernest Rutherford's work in the early 1910s, scientists' focus on radioactivity narrowed to properties of the *nuclei* within atoms. During the 1910s and 1920s, many naturally occurring radioactive processes were classified in terms of the type of radiation that was emitted during decay: α, β, or γ radiation. In time, α particles were identified

as the nuclei of Helium atoms, β particles as electrons, and γ rays as high-energy photons.[1] Nuclear scientists could then piece together typical *decay chains* among radioactive nuclei, such as

$$\begin{aligned} \mathrm{U}_{92}^{234} &\longrightarrow \mathrm{Th}_{90}^{230} + \alpha_2^4 \,, \\ \mathrm{Th}_{90}^{230} &\longrightarrow \mathrm{Ra}_{88}^{226} + \alpha_2^4 \,, \\ \mathrm{Ra}_{88}^{226} &\longrightarrow \mathrm{Rn}_{86}^{222} + \alpha_2^4 \,. \end{aligned} \tag{1}$$

In this case, a nucleus of a particular isotope of uranium (U), with 92 protons and a total atomic mass of 234 units, could emit an α particle and transform into a different nucleus. Because the α particle included two protons, the resulting nucleus moved *two places lower* on the periodic table; its nucleus now included only 90 protons instead of 92, corresponding to the element thorium (Th). Likewise, since the α particle included a total of 4 atomic mass units, the thorium nucleus would have a mass of 230 units, compared to the 234 units of the original (radioactive) uranium nucleus. The thorium nucleus, in turn, was itself radioactive. Upon emitting an α particle it would produce a nucleus of radium (Ra) with 88 protons and atomic mass of 226 units; the radium would decay via α emission to yield a nucleus of radon (Rn), and so on. In short, each time a nucleus decayed via emission of an α particle, the resulting nucleus moved two spots *down* the periodic table.

Other nuclei underwent radioactive transformations involving the emission of β particles rather than α particles. For example, decay chains like these were also observed:

$$\begin{aligned} \mathrm{Th}_{90}^{234} &\longrightarrow \mathrm{Pa}_{91}^{234} + \beta \,, \\ \mathrm{Pa}_{91}^{234} &\longrightarrow \mathrm{U}_{92}^{234} + \beta \,. \end{aligned} \tag{2}$$

Here an isotope of thorium with 234 atomic mass units decayed into a nucleus of proactinium (Pa), which has 91 protons; and later the proactinium nucleus emitted a β particle and transformed into an isotope of uranium with 92 protons. Not long after James Chadwick identified the *neutron* in 1932, Enrico Fermi suggested that these β-decay chains involved the transformation of a neutron within the radioactive nucleus into a proton plus two very light particles that quickly escaped from the nucleus: an electron (the β particle) and a new particle dubbed the "neutrino":

$$n_0^1 \longrightarrow p_1^1 + e_{-1}^0 + \bar{\nu}_0^0 \,. \tag{3}$$

The electron wasn't truly massless, but by the 1930s it had already become clear that its mass was nearly 2000 times smaller than that of the proton. Likewise, later studies clarified

[1] For a brief and accessible review, see Emilio Segrè, *From X-Rays to Quarks: Modern Physicists and Their Discoveries* (San Francisco: W. H. Freeman, 1980), chaps. 2 and 3.

that the additional particle emitted with the electron was an *antineutrino* ($\bar{\nu}$), rather than a neutrino (ν). Like the then-hypothetical neutrino, the antineutrino was quickly understood to be electrically neutral and to have a mass even smaller than that of the electron.[2]

Given Fermi's new suggestion about the β-decay of a neutron, as in Eq. (3), the decay process for the thorium nucleus in Eq. (2) could be understood as

$$\mathrm{Th}_{90}^{234} \xrightarrow{n \to p + e + \bar{\nu}} \mathrm{Pa}_{91}^{234} + \beta + \bar{\nu}\,. \tag{4}$$

Because the (anti)neutrino was electrically neutral and had a very small mass, it escaped detection, whereas the charged β ray (an electron) could be detected with common instruments like a Geiger counter. For understanding these kinds of nuclear transformations, therefore, the masses of both the electron and the antineutrino could be neglected. The larger point was that nuclei that underwent β decay moved *up* the periodic table by one spot: a neutron within the nucleus transformed into a proton, thereby increasing the total number of protons within the nucleus by one unit.

By the early 1930s, several research groups began to study "artificial" or induced radioactivity: irradiating otherwise stable elements with some sort of radiation and inducing nuclear reactions. Marie and Pierre Curie's daughter Irène Joliot-Curie and her husband Frédéric Joliot-Curie became especially active in this area. Soon after Chadwick identified the neutron in 1932, physicists like Enrico Fermi began systematically irradiating elements with neutrons to induce radioactivity. Fermi's group marched all the way up the periodic table, finally coming to the heaviest-known element, uranium. They found that when they irradiated uranium with neutrons, they could induce radioactivity via β emission. Since the new reactions involved the emission of β rays, they assumed they had transformed the target nucleus by one step up the periodic table, much as in the naturally occurring β decays of Eq. (2). That is, Fermi and his group assumed they were measuring

$$\mathrm{U}_{92}^{238} + n_0^1 \longrightarrow \mathrm{U}_{92}^{239} \xrightarrow{n \to p + e + \bar{\nu}} \mathrm{X}_{93}^{239} + \beta + \bar{\nu}\,. \tag{5}$$

In the first step, Fermi and his group reasoned, the heavy uranium nucleus absorbed the incoming neutron, increasing its total mass by one unit but leaving its number of protons (and hence its chemical identity) unchanged. Next the neutron would undergo β-decay within the nucleus, transforming into a proton and emitting an electron and (anti)neutrino as in Eq. (3). Within the remaining nucleus, there would now be 93 protons and a total mass of 239 units. Fermi was convinced that by this process, he and his group had created

[2]See Gino Segrè and Bettina Hoerlin, *The Pope of Physics: Enrico Fermi and the Birth of the Atomic Age* (New York: Henry Holt, 2016), chaps. 14-16, and Francesco Guerra and Nadia Robotti, *The Lost Notebook of Enrico Fermi: The True Story of the Discovery of Neutron-Induced Radioactivity* (New York: Springer, 2018), chap. 6.

the first *transuranic* elements: elements with more than 92 protons in each nucleus, beyond the then-known edge of the periodic table.[3] (Hence the placeholder label "X_{93}" in Eq. (5): there was no known element beyond U_{92}. An element with 93 protons was later named "Neptunium": just as the planet Neptune is the next furthest planet from the Sun after Uranus, so the element neptunium would appear on the periodic table on place further than uranium.) This was striking news; within just four years, Fermi was awarded the Nobel Prize in Physics for this work.[4]

At least one researcher at the time — nuclear chemist Ida Noddack — raised questions about whether Fermi's group had really produced transuranic nuclei in these experiments. Drawing on her training as a chemist, she cautioned against identifying new elements of the periodic table without thoroughly testing their chemical properties, and checking for consistency with the chemical behavior of known elements within the same columns of the periodic table.[5] But Noddack's cautions were almost universally overlooked at the time. After all, Fermi's interpretation of his group's experiments was very much in keeping with decades of experience among nuclear scientists by that time: nuclear transformations typically moved an element up the periodic table by one spot (β decay) or down the periodic table by two spots (α decay).

Nuclear Fission

Other groups began repeating Fermi's induced-radioactivity experiments, irradiating many different elements with neutrons. A group in Berlin, including nuclear chemists Otto Hahn and Fritz Strassmann and nuclear physicist Lise Meitner, became especially active in this area. In the midst of those experiments — during July 1938 — Meitner was forced to flee Germany. She was an Austrian citizen of Jewish background, so she only became subject to the Nazis' anti-Semitic employment laws after the *Anschluss* of March 1938, which formally incorporated Austria within the German Reich.[6]

Unlike Fermi, Hahn and Strassmann were trained as chemists, and they were especially adept at performing chemical analyses of the reaction products. In one of the last experi-

[3]E. Fermi, "Possible production of elements of atomic number higher than 92," *Nature* **133** (1934): 898-899; E. Amaldi, O. D'Agostino, E. Fermi, B. Pontecorvo, F. Rasetti, and E. Segré, "Artificial radioactivity produced by neutron bombardment, II," *Proceedings of the Royal Society London* **A149** (1935): 522-558.

[4]Segrè and Hoerlin, *Pope of Physics*, chaps. 17-18; Guerra and Robotti, *Lost Notebook of Enrico Fermi*, chaps. 7-10.

[5]I. Noddack, "Über das Element 93," *Zeitschrift für Angewandte Chemie* **47** (1934): 653-655; I. Noddack, "Das Periodische System der Elemente und Seine Lücken," *Zeitschrift für Angewandte Chemie* **47** (1934): 301-305. See also Gildo Magalhães Santos, "A tale of oblivion: Ida Noddack and the 'universal abundance' of matter," *Notes and Records of the Royal Society* **68** (2014): 373-389.

[6]See esp. Ruth Lewin Sime, *Lise Meitner: A Life in Physics* (Berkeley: University of California Press, 1996), chaps. 6-10.

ments that the Berlin group conducted before Meitner was forced to flee, they had found that the reaction products following bombardment of uranium with neutrons behaved *chemically* in a similar way to barium. Barium, with atomic number 56, was nowhere near uranium (atomic number 92) on the periodic table, but it was exactly one row above radium (atomic number 88), and hence barium and radium could be expected to behave similarly in chemical reactions. So the Berlin group had concluded, in their May 1938 experiment, that the irradiated uranium nuclei (perhaps by way of some unstable intermediary) eventually underwent a short α-decay chain, similar to that of Eq. (1), to produce radium among the reaction products.[7]

Meitner convinced Hahn to re-do the experiments with uranium and to conduct more thorough chemical tests of the reaction products. Hahn and Strassmann were able to complete the new experiments after Meitner's forced departure.[8] They became convinced that they were *not* producing a transuranic element, one step beyond uranium on the periodic table, which might have then undergone typical α decays to arrive at radium. Rather, they found clear evidence of barium itself, not just something chemically similar to barium. Yet barium was almost half the size of uranium — indicating a huge transition from the nuclear reaction, down dozens of spots on the periodic table. Such a huge leap had never been identified before; all known nuclear transformations had involved small steps up or down the periodic table. Hahn and Strassmann hastily wrote up their results in December 1938, suggesting that experiments like theirs and Fermi's had actually corresponded to

$$\mathrm{U}_{92} + n_0 \longrightarrow \mathrm{Ba}_{56} + \mathrm{Kr}_{36}\,. \tag{6}$$

They acknowledged just how shocking such a reaction would be. In their closing paragraph, they wrote, "As chemists, we must actually say the new particles do not behave like radium but, in fact, like barium; as nuclear physicists, we cannot make this conclusion, which is in conflict with all experience in nuclear physics."[9]

Meitner had narrowly escaped Nazi Germany during the summer of 1938; she was given a temporary position at a physics institute associated with the Swedish Academy of Sciences in Stockholm. In December 1938 she met up with her nephew, the young physicist Otto

[7]L. Meitner, F. Strassmann, and O. Hahn, "Künstliche Umwandlungsprozesse bei Bestrahlung des Thoriums mit Neutronen; Auftreten isomer Reihen durch Abspaltung von α-Strahlen," *Zeitschrift für Physik* **109** (1938): 538-552; see also Elisabeth Crawford, Ruth Lewin Sime, and Mark Walker, "A Nobel tale of postwar injustice," *Physics Today* **50** (September 1997): 26-32.

[8]Crawford, Lewin Sime, and Walker, "Nobel tale," 26.

[9]O. Hahn and F. Strassmann, "Über den Nachweis und das Verhalten der bei der Bestrahlung des Urans mittels Neutronen entstehenden Erdalkalimetalle," *Naturwissenschaften* **27** (1939): 11-15, on 15: "Als Chemiker müssten wir aus den kurz dargelegten Versuchen das oben debrachte Scheme eigentlich umbenennen und statt Ra, Ac, Th die Symbole Ba, La, Ce einsetzen. Als der Physik in gewisser Weise nahestehende 'Kernchemiker' können wir uns zu disem, allen bisherigen Erfahrungen der Kernphysik widersprechenden, Sprung noch night entschliessen."

Robert Frisch, for a ski holiday in Sweden. At the time Frisch was a postdoctoral researcher at Niels Bohr's Institute for Theoretical Physics in Copenhagen. Just before Meitner's and Frisch's trip, Meitner received a letter from Hahn with an update about the latest Berlin tests and their detection of barium among the reaction products. As Frisch later recalled, "When I came out of my hotel room after my first night in [the ski village] Kungälv I found Lise Meitner studying a letter from Hahn and obviously worried about it. I wanted to tell her of a new experiment I was planning, but she wouldn't listen; I had to read that letter. Its content was indeed so startling that I was at first inclined to be sceptical." They continued to puzzle through the implications of Hahn's letter throughout the day. At one point, Frisch recalled, "we both sat down on a tree trunk (all that discussion had taken place while we walked through the wood in the snow, I with my skis on, Lise Meitner making good her claim that she could walk just as fast without), and started to calculate on scraps of paper. [...] The uranium nucleus might indeed resemble a very wobbly, unstable drop, ready to divide itself at the slightest provocation, such as the impact of a single neutron."[10] While trudging through the snow that day — and occasionally pausing on a bench or tree trunk — Meitner and Frisch worked out the first-ever physical explanation of nuclear fission.

They began by reasoning that large nuclei, like uranium, must be barely stable.[11] Perhaps the attractive nuclear force among protons and neutrons that kept atomic nuclei from falling apart was just barely able to compensate for the electrostatic (Coulomb) repulsion of nearly 100 positively charged protons closely packed within a nucleus, each repelling the others. In that case, if the nucleus were *perturbed* by an incoming neutron — especially if that neutron had been *slowed down* to a low energy, thereby stretching out its quantum-mechanical de Broglie wavelength, $\lambda = h/(mv)$ — then the entire, unsteady nucleus could begin to shake or wobble like a liquid drop. The drop could then divide into two drops, each of roughly equal size and in close proximity to each other. Following division (or "fission"), there would now be two globs of nuclear matter, each filled with dozens of protons and exerting a strong electrostatic repulsion upon the other. Hence the two fission products should each quickly gain a large amount of kinetic energy, as they raced apart from each other.

Meitner and Frisch first estimated the energy scales involved. Prior to fission, a large nucleus like uranium would have a nuclear energy $E_{\rm nuc}$ approximately balancing the electrostatic energy arising from each of those protons in the nucleus repelling the others. They

[10] Otto Robert Frisch, *What Little I Remember* (New York: Cambridge University Press, 1979), on pp. 115-116.

[11] The following discussion fills in the intermediate steps which Meitner and Frisch left tacit in their first, brief report about their new work: L. Meitner and O. R. Frisch, "Disintegration of uranium by neutrons: A new type of nuclear reaction," *Nature* **143** (1939): 239-240.

could therefore estimate

$$E_{\rm nuc} \simeq \sum_{i,j=1,\, i\neq j}^{q} \frac{n_i\, n_j\, e^2}{r_{ij}}\,. \tag{7}$$

That is, they could sum up the energy associated with the electrostatic repulsion of each proton (labeled n_i and possessing electric charge e) from every other proton (labeled n_j and charge e), up to the total q protons contained with the nucleus; each pair $n_i\, n_j$ was separated by some distance r_{ij}. We can think of the expression $n_i\, n_j e^2/r_{ij}$ as a large $q \times q$ square matrix, each element of which represents the energy associated with the electrostatic repulsion of a particular pair of protons. A square $q \times q$ matrix includes q^2 elements. As indicated in the sum by the notation $i \neq j$, they would not include the q entries along the diagonal of the matrix, with $n_i = n_j$; those would correspond to the repulsion of proton n_i from itself. Hence the number of elements from the matrix to include is $q^2 - q$. For $q \gg 1$, this is approximately equal to q^2.

To simplify the problem — after all, Meitner and Frisch were interested in a rough order-of-magnitude estimate — they next assumed that each of the pairs of protons (n_i, n_j) was separated by an average or typical distance $r_{ij} \sim R_{\rm nuc}$, roughly the size of the uranium nucleus. In that case, the expression in Eq. (7) could be simplified to

$$E_{\rm nuc} \sim \frac{(qe)^2}{R_{\rm nuc}}\,. \tag{8}$$

For a uranium nucleus, they could estimate $q \sim 100$, close to the actual number of 92 protons within the nucleus. For the typical size $R_{\rm nuc}$ of a uranium nucleus, they could reason as follows. Recall that the Bohr radius for the ground-state of an electron in a hydrogen atom is $a_0 \simeq 5.3 \times 10^{-9}$ cm, and that Rutherford's scattering experiments suggested that the radius of an atomic nucleus was typically about 10^5 smaller than the radius of the corresponding atom. Meitner and Frisch estimated that the radius of a uranium atom should be at least 100 times larger than the radius of a hydrogen atom, and hence, in round numbers, they could estimate

$$R_{\rm nuc} \sim \underbrace{10^{-9}\,{\rm cm}}_{\text{radius of a hydrogen atom}} \times \underbrace{10^{2}}_{\text{ratio of U to H atomic radii}} \times \underbrace{10^{-5}}_{\text{ratio of nuclear to atomic radii}} \sim 10^{-12}\,{\rm cm}\,. \tag{9}$$

Chemical reactions, on the other hand, typically involved the transfer of a single (valence) electron from one atom to another across some distance $R_{\rm atom}$, and hence Meitner and Frisch could estimate

$$E_{\rm chem} \sim \frac{e^2}{R_{\rm atom}}\,. \tag{10}$$

Atoms larger than the ground-state of hydrogen would have typical radii $R_{\rm atom} \sim 10^{-8}$ cm. Comparing Eqs. (8) and (10), using $q \sim 100$, $R_{\rm nuc} \sim 10^{-12}$ cm, and $R_{\rm atom} \sim 10^{-8}$ cm yielded

a ratio

$$\frac{E_{\rm nuc}}{E_{\rm chem}} \sim \left(\frac{(qe)^2}{R_{\rm nuc}}\right)\left(\frac{R_{\rm atom}}{e^2}\right) \sim (100)^2 \left(\frac{10^{-8}\,{\rm cm}}{10^{-12}\,{\rm cm}}\right) \sim 10^8\,. \tag{11}$$

In other words, the typical energies involved in nuclear reactions among very large nuclei, such as uranium, should be as much as *one hundred million times larger* than typical energies involved in chemical reactions!

By the late 1930s, nuclear physicists typically measured the energies of various reactions in units of electron-Volts, or eV. Recall that the ionization energy of a hydrogen atom — that is, the energy required to remove its single electron — is 13.6 eV; chemical reactions typically involve energies in the range $E_{\rm chem} \sim \mathcal{O}(1{-}10\,{\rm eV})$. Meitner and Frisch's back-of-the-envelope calculation suggested that typical energies involved in nuclear reactions should instead be $E_{\rm nuc} \sim \mathcal{O}(10^8 - 10^9\,{\rm eV})$, that is, hundreds of millions of electron-Volts, $\mathcal{O}(10^2\,{\rm MeV})$, up to billions of electron-Volts, $\mathcal{O}(1\,{\rm GeV})$. This was an enormous shift in energy scale.

Meitner and Frisch weren't done. Their next step was to consider how much energy might typically be *released* each time a whole uranium nucleus underwent fission. Prior to fission, the nucleus would have energy

$$E_{\rm whole} \sim E_{\rm nuc} \sim \frac{(qe)^2}{R_{\rm nuc}}\,, \tag{12}$$

with $q \sim 100$. After fission, two smaller nuclei would remain, each with roughly equal numbers of charges $q/2$ packed into roughly equal *volumes*, $V_{\rm piece}$:

$$V_{\rm piece} = \frac{1}{2} V_{\rm whole}\,. \tag{13}$$

The volume of a sphere scales as $V \sim R^3$, or $R \sim V^{1/3}$, so Meitner and Frisch could estimate

$$\frac{R_{\rm piece}}{R_{\rm whole}} = \left(\frac{V_{\rm piece}}{V_{\rm whole}}\right)^{1/3} = \left(\frac{1}{2}\right)^{1/3} \simeq 0.8\,, \tag{14}$$

where $R_{\rm whole} = R_{\rm nuc}$, the radius of the whole (original) uranium nucleus prior to fission. That suggested that following fission, the energy of each piece, compared to the energy of the whole (original) uranium nucleus, should be given roughly by

$$\frac{E_{\rm piece}}{E_{\rm whole}} \sim \left(\frac{\left(\frac{q}{2}e\right)^2}{0.8\,R_{\rm nuc}}\right)\left(\frac{R_{\rm nuc}}{(qe)^2}\right) = \left(\frac{1}{2}\right)^2\left(\frac{1}{0.8}\right) \simeq 0.3 \sim \frac{1}{3}\,. \tag{15}$$

In other words, following fission each piece would acquire about *one-third* of the original energy of the uranium nucleus, $E_{\rm piece} \sim E_{\rm whole}/3$, with $E_{\rm whole} \sim E_{\rm nuc} \sim \mathcal{O}(10^2 - 10^3\,{\rm MeV})$. That result, in itself, was remarkable: a single incoming slow neutron, with kinetic energy $\sim \mathcal{O}(1\,{\rm eV})$, could split a heavy nucleus and produce two fission fragments, each with a kinetic

energy up to one hundred million times greater than the energy injected into the system by the neutron! Not only that, but the energy carried off by the fission fragments would only account for a fraction of the total energy released. An additional amount of energy

$$\Delta E = E_{\rm whole} - 2E_{\rm piece} \sim \frac{1}{3} E_{\rm nuc} \tag{16}$$

would also be released as *raw energy*, each time a single uranium nucleus underwent fission.

Meitner and Frisch quickly wrote a brief Letter to the Editor of *Nature* describing their new physical model of nuclear fission; it was received at the journal on January 16, 1939 and published in the February 11 issue. In addition to describing the basic physical mechanism, they described their simple calculation, as sketched in these notes, and predicted that the fission fragments should each acquire typical kinetic energies of about 10^2 MeV.[12] As soon as he returned to Bohr's Institute, Frisch was also able to conduct new laboratory measurements, which quickly confirmed exactly this energy scale. Soon after that, researchers in other laboratories — both in Britain and in Germany — independently measured comparable energies in their own new experiments.[13]

Frisch later recalled the swirl of events: he "rigged up" equipment in the basement of Bohr's Institute over the course of a few days, "and then I worked most of the night to do the measurements because the counting rates were very low. But by three in the morning I had the evidence of the big pulses [corresponding to the energetic fission fragments]. And I went to bed at three in the morning, and then at seven in the morning I was knocked out of bed by the postman who brought a telegram to say that my father had been released from the concentration camp."[14]

Frisch had given a copy of the short paper that he and Meitner had just prepared for *Nature* to his supervisor, Niels Bohr, days before Bohr set off for a trip to the United States in January 1939. As soon as Bohr arrived for his sabbatical visit at the Institute for Advanced Study in Princeton, he began working with his colleague (and former postdoctoral advisee) John Wheeler; Wheeler was by then a professor of physics at nearby Princeton University.[15] They formalized Meitner's and Frisch's physical model of nuclear fission, which had built

[12] Meitner and Frisch, "Disintegration of uranium by neutrons," on p. 239. Within a few weeks, they published a brief follow-up article as well: L. Meitner and O. R. Frisch, "Products of fission of the uranium nucleus," *Nature* **143**: 471-472, received at the journal on 6 March 1939 and published in the 18 March issue.

[13] O. R. Frisch, "Physical evidence for the division of heavy nuclei under neutron bombardment," *Nature* **143** (1939): 276. See also R. D. Fowler and R. W. Dodson, *Nature* **143** (1939): 233; and W. Jentschke and F. Prankl, "Untersuchung der schweren Kernbruschstücke beim Zerfall von neutronenbestrahltem Uran und Thorium," *Naturwissenschaften* **27** (1939): 134-135.

[14] A brief excerpt from a recording of an interview with Otto Robert Frisch, from which this quotation is taken, is available as part of the online exhibit "The discovery of fission," available at `https://history.aip.org/history/exhibits/mod/fission/fission1/04.html` (accessed 28 September 2020).

[15] Niels Bohr and John A. Wheeler, "The mechanism of nuclear fission," *Physical Review* **56** (1939): 426-450.

upon Bohr's own (previous) liquid-drop model of large nuclei. Bohr and Wheeler also showed that Meitner and Frisch's initial estimate — that the energy released during the fission process should be of order $E_{\rm nuc} \sim \mathcal{O}(10^2 - 10^3\,{\rm MeV})$ — was consistent with what one could estimate using Einstein's by-then famous formula, $E = mc^2$. In particular, Bohr and Wheeler considered the *nuclear binding energy* for the uranium nucleus prior to fission, and for the fission products following the reaction.

The fission reaction of Eq. (6) typically involved specific isotopes of U, Ba, and Kr, as well as the release of a few excess neutrons:

$$\mathrm{U}_{92}^{235} + n_0^1 \longrightarrow \mathrm{Ba}_{56}^{141} + \mathrm{Kr}_{36}^{92} + 3n_0^1\,. \tag{17}$$

Here we can see that both the total number of protons balances before and after the reaction (92), and the total number of atomic mass units appears to balance (236). However, the actual mass of a nucleus is *less* than the sum of its constituent parts; the difference became known as the "mass defect," which could be directly related to the nuclear binding energy by means of Einstein's relation between energy and mass. Using modern values but following the type of reasoning established in Bohr and Wheeler's paper, we may consider the mass of a proton (m_p) and a neutron (m_n) in atomic mass units (amu)[16]:

$$m_p = 1.0073\,{\rm amu}\,,\;\; m_n = 1.0087\,{\rm amu}\,. \tag{18}$$

The mass of the nuclear constituents within the uranium nucleus prior to fission is therefore

$$m_{\rm U,sum} = 92m_p + (235 - 92)m_n = 236.9157\,{\rm amu}\,. \tag{19}$$

The actual, measured mass of a U_{92}^{235} nucleus, on the other hand, is

$$m_{\rm U,meas} = 235.0439\,{\rm amu}\,. \tag{20}$$

For this isotope of uranium, the mass defect is therefore given by

$$\Delta m_{\rm U} = m_{\rm U,meas} - m_{\rm U,sum} = -1.8718\,{\rm amu}\,. \tag{21}$$

Using Einstein's relation, this mass defect can be expressed as a nuclear binding energy:

$$E_{\rm U,bind} = \Delta m_{\rm U} c^2 = (-1.8718\,{\rm amu})\left(\frac{931.5\,{\rm MeV}/c^2}{1\,{\rm amu}}\right)c^2 = -1743.58\,{\rm MeV}\,, \tag{22}$$

upon converting from atomic mass units to the mass-scale ${\rm MeV}/c^2$. (The binding energy in general is *negative*, since it is associated with an *attractive* nuclear force; this is what

[16]The numerical values used here may be found in the online edition of John R. Rumble, ed., *CRC Handbook of Physics and Chemistry*, 101st edition (Boca Raton, FL: CRC Press / Taylor & Francis, 2020), available via the MIT Libraries website.

must compensate for the positive energy associated with the protons' electrostatic repulsion within the nucleus.) Note that this calculation of the energy scale associated with the uranium nucleus prior to fission is consistent with Meitner and Frisch's estimate of $E_{\rm nuc} \sim \mathcal{O}(10^2 - 10^3\,{\rm MeV})$.

Proceeding similarly, we may compute the mass defects and nuclear binding energies for the fission products:

$$\begin{aligned} m_{\rm Ba,sum} &= 56 m_p + (141-56) m_n = 142.1483\,{\rm amu}, \\ m_{\rm Ba,meas} &= 140.9144\,{\rm amu}\,, \\ \Delta m_{\rm Ba} &= m_{\rm Ba,meas} - m_{\rm Ba,sum} = -1.2339\,{\rm amu}\,, \\ E_{\rm Ba,bind} &= -1149.38\,{\rm MeV} \end{aligned} \tag{23}$$

and

$$\begin{aligned} m_{\rm Kr,sum} &= 36 m_p + (92-36) m_n = 92.7500\,{\rm amu}, \\ m_{\rm Kr,meas} &= 91.9262\,{\rm amu}\,, \\ \Delta m_{\rm Kr} &= m_{\rm Kr,meas} - m_{\rm Kr,sum} = -0.8238\,{\rm amu}\,, \\ E_{\rm Kr,bind} &= -767.37\,{\rm MeV}\,. \end{aligned} \tag{24}$$

Comparing the nuclear binding energies before and after fission, we then find

$$\begin{aligned} \Delta E &= E_{\rm U,bind} - (E_{\rm Ba,bind} + E_{\rm Kr,bind}) \\ &= -1743.58\,{\rm MeV} - (-1149.38\,{\rm MeV} - 767.37\,{\rm MeV}) \\ &= +173.17\,{\rm MeV}\,. \end{aligned} \tag{25}$$

In other words, there is an *excess* of energy available following the fission reaction compared to before fission: the amount of nuclear binding energy required to keep the uranium nucleus bound together prior to fission is *greater* than the nuclear binding energy required to keep the barium and krypton fission products bound together following the reaction. That excess binding energy gets released by the fission reaction. And note that the energy scale — even when using modern values — is remarkably consistent with Meitner's and Frisch's original estimate, $\sim \mathcal{O}(10^2 - 10^3\,{\rm MeV})$, even though they had approached the calculation in an entirely different way.

STS.042J / 8.225J Einstein, Oppenheimer, Feynman: Physics in the 20th Century
Fall 2020