The Double-Slit Experiment: An Adventure in Three Acts

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March 2011

The double-slit experiment throws into stark relief two of the most enduring enigmas about quantum mechanics: the role of probabilities, and the strange intermixing of particle and wave concepts ("wave-particle duality"). We will begin by considering two separate classical scenarios: firing macroscopic bullets at a wall, and watching an ocean wave pass through a coral reef. Then we will turn to the inherently quantum-mechanical situation of shooting electrons at a barrier, such as a crystal lattice.

Act I: Classical particles

Imagine you are firing bullets at a bullet-proof wall that has two narrow slits in it, labeled *A* and *B*, through which bullets may pass. The slits are a distance *d* apart from each other. (See Fig. 1.)

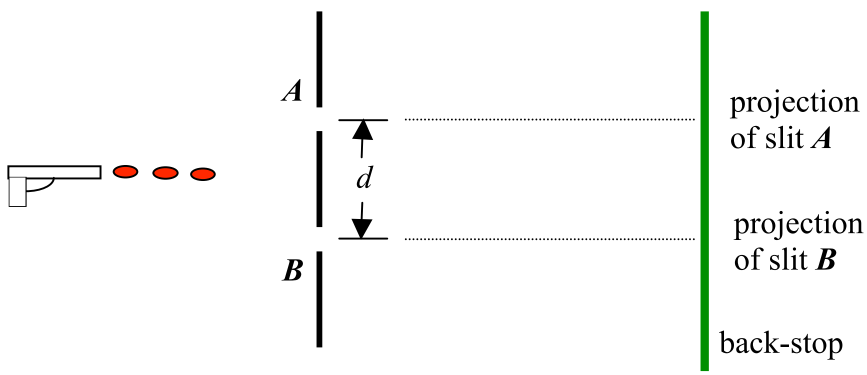

FIG 1. Firing macroscopic bullets at a wall with two slits in it. The bullets will be collected at the back-stop, and their locations noted.

After you fire your rounds at the wall, your assistant counts the number of bullets that arrived at a back-stop (behind the wall-with-slits). She makes a histogram showing the number of bullets that arrived as a function of position. For classical particles like bullets, this is one way of measuring the *probability distribution* of where any individual bullet would most likely be found along the back-stop.

For your first round of shooting you cover slit *B*, so that only slit *A* is open. After you have fired ten thousand rounds at the wall, your assistant produces a graph showing the number of bullets found versus position on the back-stop, making special note of the projected positions of slits *A* and *B*. Her results are shown on the left in Fig. 2. As expected, the bullets that made it through the barrier are clustered in a bell-shaped curve, with a peak directly behind the open slit.

Being a conscientious experimenter, you repeat the exercise, this time covering slit *A* so that only slit *B* is open. Once again you fire ten thousand rounds, after which your assistant produces a histogram showing the new results (right in Fig. 2.).

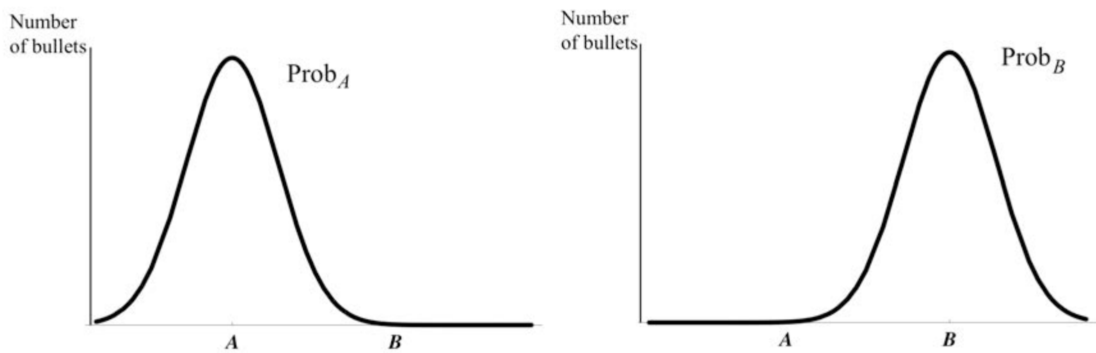

FIG. 2. Histograms showing number of bullets as a function of position along the back-stop, when only one slit is open. The projected positions of slits *A* and *B* are shown. These graphs can be interpreted as giving the *probability distributions*, Prob_A and Prob_B, of where any given bullet would be found when fired toward the wall when only one slit (*A* or *B*, respectively) is open.

Now you open up both slits *A* and *B* and fire yet another ten thousand rounds at the barrier. When the dust has settled, your tireless assistant produces the histogram shown in Fig. 3.

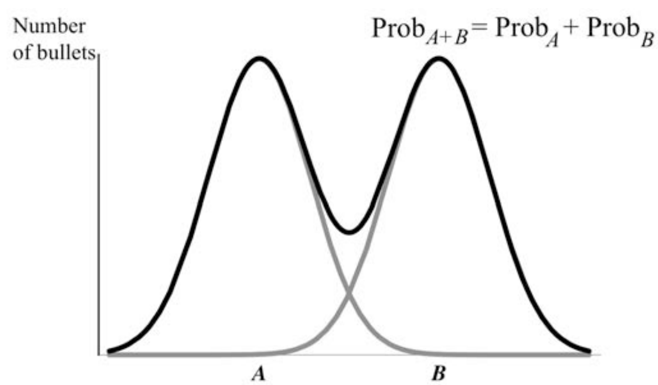

FIG. 3. Histogram showing number of bullets as a function of position along the back-stop, when both slits are open. The black line shows the final results; the gray lines show the previous results when only slit *A* or *B* was open.

After her exhaustive counting, your assistant concludes that the bullets from this last round fall into a bimodal distrubtion: one peak located directly behind slit *A*, and another peak directly behind slit *B*. In fact, as she later confirms, the probability of finding a bullet at a particular location is just the *sum* of the probabilities for each of the separate single-slit distributions. When probabilities simply add in this way, the underlying processes must be *independent* of each another. In this case, that means that each individual bullet either traveled through slit *A* or through slit *B*—two independent possibilities.

Act II. Classical waves

Because you have a large research grant and a penchant for travel, you leave your rifle behind and journey to Australia for your next experiments. Your new goal: to study the behavior of water waves when they pass through gaps in the Great Barrier Reef. In particular, you find a large coral reef that effectively blocks incoming waves from reaching the shore, *except* for two small gaps in the reef through which waves may pass. While you observe the experiment from your yacht, your intrepid assistant rows to shore. Her job is to measure the intensity of the ocean waves that reach the shore after passing through the reef.

Having studied hydrodynamics, you know that the intensity of the wave upon reaching the shore will be given by the absolute square of the wave's amplitude. (The same holds true in optics: the intensity of light goes as the absolute square of the amplitudes of the electric and/or magnetic fields making up the light wave.) For convenience, you label the ocean wave's amplitude by the letter φ; the wave's intensity will then be given by $I = |\varphi|^2$.

For your first study, you plug one of the two holes in the reef, so that waves can only pass through the gap labeled *A*. After the waves have passed through the single-gap reef, your assistant plots the intensity of the wave that reaches shore. Then you repeat the test, plugging gap *A* and opening gap *B*. As before, your assistant is careful to mark the projected position of gaps *A* and *B* along the shoreline. Her results are shown in Fig. 4.

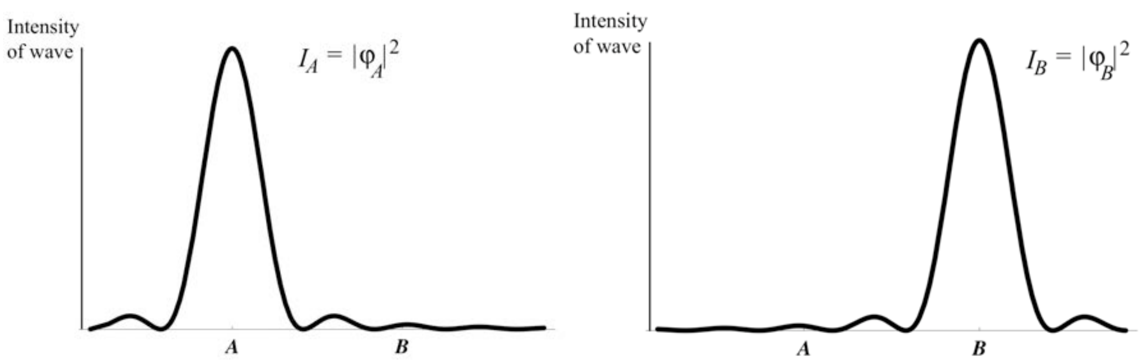

FIG. 4. Intensity of the ocean wave upon reaching the shore as a function of position along the shoreline, when only one gap is open. Aside from a series of small diffraction peaks of diminishing magnitude, the intensity is dominated by a broad central peak.

When only one gap is open in the coral reef, the ocean waves reaching the shore have a greatest intensity directly behind the reef gap. In fact, apart from rather tiny diffraction peaks on either side of the broad central peak (the greatest of whose heights is typically only a few percent as high as the central maximum), the wave intensity looks remarkably like the probability distributions pictured in Fig. 2.

Next you open both gaps *A* and *B*. Now your assistant sees quite an amazing pattern on the shore. (See Fig. 5.)

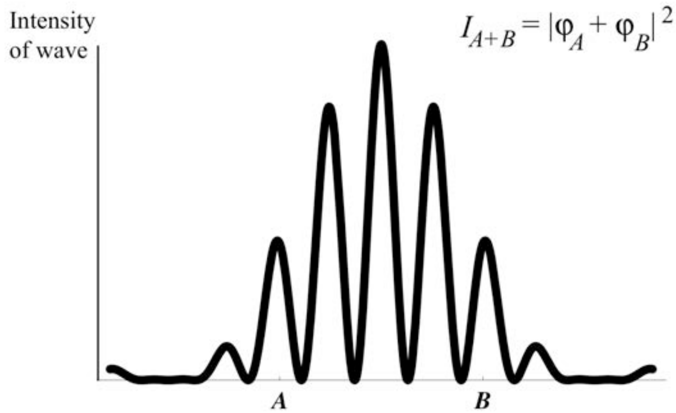

FIG. 5. Intensity of the ocean wave upon reaching the shore when both gaps A and B are open.

Hardly the sum of the two previous intensity patterns, the ocean waves have *interfered* with one another. The waves passing through gap A get superposed with waves passing through gap B, yielding a new amplitude, $(\varphi_A + \varphi_B)$. At some places along the shoreline, the crest of one wave arrives with the trough of another, leading to destructive interference and no wave intensity. In other places, crest arrives with crest, leading to constructive interference and a high peak in intensity. In fact, the region with greatest intensity is now directly between the projected positions of the two gaps, even though neither single-gap intensity pattern showed much of a peak there at all.

Rather than seeing two broad central peaks, your assistant thus measures a series of narrow peaks separated by regions along the shore with virtually no wave intensity at all. The intensity pattern measured on the shore is not simply the sum of two separate single-gap intensities. Unlike the case of the bullets being fired at the wall, in other words, the ocean waves are not independent of each other at all.

Act III. Quanta

Well-rested from your adventure ‘down under,’ you return to your laboratory. You plan to conduct one final round of experiments. You have heard a little something about this thing called “quantum mechanics,” and you decide to see what the big fuss is all about. So this time your quarry is electrons rather than bullets or ocean waves.

You fire electrons at a tiny barrier manufactured by your friendly neighborhood nanotechnology start-up company. Like before, there are only two gaps through which the electrons may pass. In passing, you heard something about matter behaving like waves, which you thought sounded quite foolish. To show just how foolish, you fix the distance between the gaps in your new nanobarrier, d, to be ten thousand times larger than the so-called de Broglie wavelength of the electrons you are using. Being a stickler, you also decide the fire electrons at the barrier one at a time. You wait a full hour in between each of these single-electron runs, just to make sure there won’t be any interactions between electrons while in flight. Behind the barrier you have placed a tight array of hypersensitive detectors, so that you will be able to measure the location of any electrons that pass through the gaps in the barrier.

(Being ignorant of history, by the way, you are doomed to repeat it. The experiments you have decided to undertake are basically a repeat of those done in 1927 by Clinton Davisson and Lester Germer at Bell Labs, who fired electrons at a crystal lattice to see where they would come out. In fact, your experiment also borrows a key feature from another 1927 experiment, by A. J. Dempster and H. F. Batho of the University of Chicago, who sent only one light-quantum through their apparatus at a time to make sure that the interference effects could not be accounted for by interactions between separate quanta.[1])

As before, you block one of the gaps so that electrons may only pass through slit *A*. Your assistant plots a histogram of where on the back wall of detectors individual electrons eventually get registered. Every single electron that you fire gets registered as a satisfyingly discrete, localized little particle—just as you suspected they would be. (So much for this harebrained "wave-particle duality," you conclude. Silly theorists…) After ten thousand electrons have been fired—one at a time—toward the barrier, your assistant presents her tally. She insists on describing her results in terms of some "wavefunction," ψ, the square of which gives the probability distribution for where electrons will be found. "Fine," you say, "you're just counting, anyway." Her results are shown in Fig. 6.

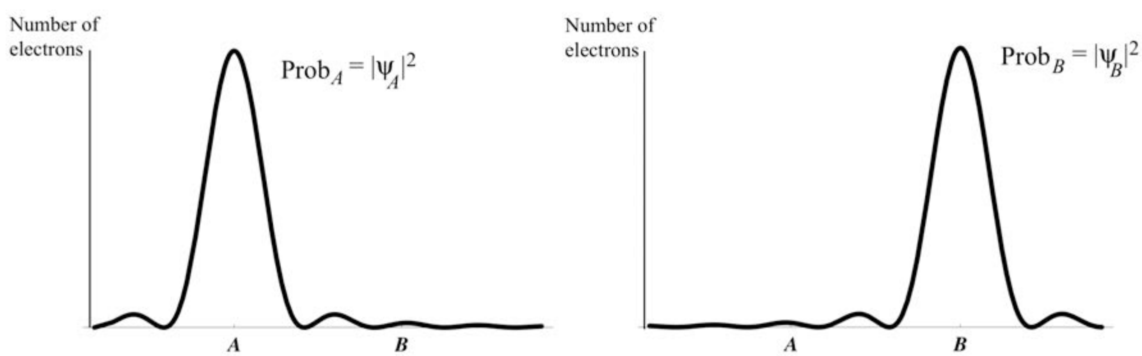

FIG. 6. Histogram of electron detections as a function of position, when only one slit is open.

Being a good experimentalist, you easily dismiss the spurious peaks to the left and right of the central peak. These weigh in at the percent level, and you have a suspicion that your detectors have some residual systematic error of about the same amount. So you happily conclude that electrons behave just like tiny bullets after all: fire them at a slit, and each one will be detected like a tiny bullet. Moreover, they will clump directly behind the open slit. What's the big deal?

Now you open both slits and fire ten thousand more electrons at the barrier, again waiting one hour after each individual electron has been released. Your smug assistant (who has studied quantum mechanics and knew what answer to expect in the first place) shows you her tally (see Fig. 7.)

[1] C. Davisson and L. H. Germer, "Diffraction of electrons by a crystal of nickel," *Physical Review* 30 (1927): 705-740; A. J. Dempster and H. F. Batho, "Light quanta and interference," *Physical Review* 30 (1927): 644-648.

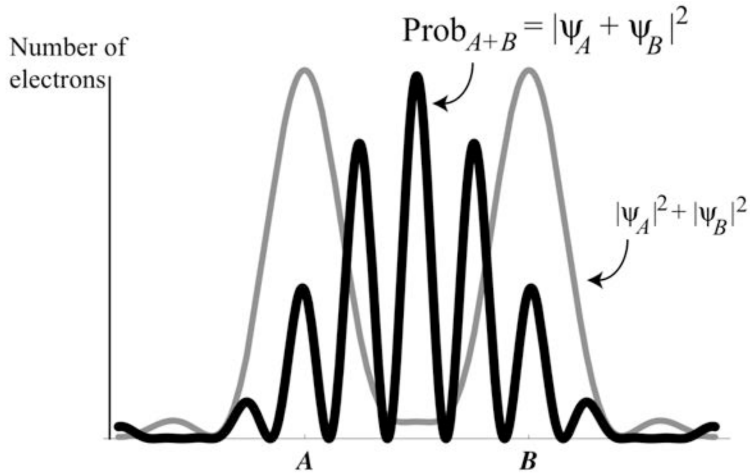

FIG. 7. Histogram showing number of electrons detected as a function of position, when both slits are open. The final results are shown in black. The gray line shows, for comparison, the sum of the two single-slit probability distributions. Unlike the case of classical particles, in the quantum-mechanical case the final distribution does not equal the sum of the two single-slit distributions.

Try as you might, you simply can't make sense of this in terms of classical particle behavior. Instead, you realize quite soon that you've seen just this sort of pattern before: in your experiment with interfering classical ocean waves.

But how could that be? Each electron has been sent through one at a time; each electron was detected as a single localized, tiny little particle—none showed up as a washed-out extended wave, like the ocean wave on shore. And yet after ten thousand individual quanta have been sent through the apparatus, the result looks exactly as if two waves had been superposed and undergone constructive and destructive interference. What did the waving? Not any individual electron—after all, you designed your experiment so that the electrons' de Broglie wavelength was ten thousand times smaller than the gap between the slits, so it hardly makes sense that one single electron went through both holes and interfered with itself on the other side. Instead, the *probability amplitude*, ψ, spread out like a wave and went through both slits. And, as your assistant had long ago learned from Max Born, the probability for detecting a quantum object at a given place at a specific time is given by the *square* of this probability amplitude, Prob = $|\psi|^2$.

So you started out treating electrons like tiny particles—and with good reason, given the way each one was detected at the end—and yet you also needed to draw on wave ideas, such as superposition and interference, to make sense of your empirical data. Were the electrons acting like particles or like waves? Yes: all of the above.

Epilogue: A Slit Detector?

Still unsatisfied by this wave-particle business, you decide to modify your apparatus to allow you to measure through which slit an individual electron passes. After all, you know in your bones that electrons behave like tiny little particles; and so each electron simply *must* have passed through one or the other slit, just as the bullets had each done in your first set of experiments. So you place some tiny test particles behind the slits.

As before, electrons will be fired from the source *S* toward the barrier. If an electron passes through slit *A* en route to the detectors, then you should see some of these test particles get scattered, just like pins tossed by a bowling ball: this would be a positive confirmation that the electron went through slit *A* on that round. If, on the other hand, none of your test particles gets scattered, then the electron must have passed through slit *B*. (See Fig. 8.)

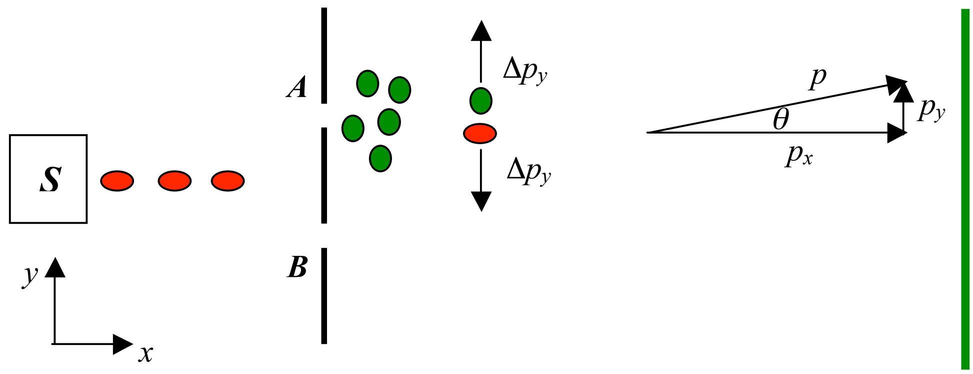

FIG. 8. Modified apparatus, with test particles behind slit *A*. If an electron passes through slit *A*, it will scatter with one of the test particles, after which both particles will have some uncertainty in their momentum along the *y* direction, Δp_y. Also shown is the momentum for an unscattered electron.

In order for your test particles to yield a good measurement of whether or not the incoming electron passed through a given slit, you need the uncertainty in the test particles' position to be small: $\Delta y << d$, where Δy is the uncertainty in their position (in the *y* direction), and *d* is the distance between slits. Meanwhile, the collision between the electron and the test particles will produce an uncertainty in the electron's momentum along the *y*-direction, Δp_y, and an equal but opposite uncertainty in the *y*-component of the test particles' momenta.

Consider the effect of that uncertainty, Δp_y, on the interference pattern at the back screen. For classical waves, the position, *P*, of the first minimum of the interference pattern—that is, the location on the back screen nearest the central peak at which crests of one wave arrive with troughs of another to cancel each other out in destructive interference—occurs where the difference in path lengths from the two slits is equal to a half-integer number of wavelengths. (See Fig. 9.)

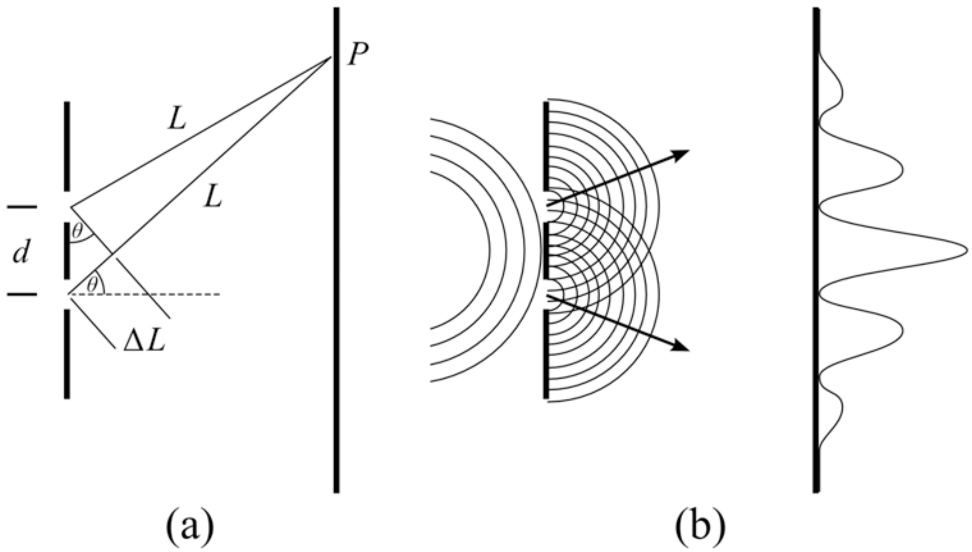

FIG. 9. Interference of waves passing through a double slit. Image by Alex Wellerstein.

The difference in path lengths, ΔL, from each slit to point P is given by $\Delta L = d \sin \theta$. The first minimum in the interference pattern, nearest the central peak, therefore occurs for $\Delta L = (1/2)\, \lambda$, where λ is the wavelength of the waves passing through the slits, or

$$d \sin\theta = \frac{1}{2}\lambda \ ,$$

corresponding to an angle from the initial direction of propagation given by

$$\sin\theta = \frac{\lambda}{2d} \ .$$

An electron's associated wavelength is given by the de Broglie relation,

$$\lambda = \frac{h}{p} \ ,$$

where h is Planck's constant and p is the electron's momentum. Thus the minima in the interference pattern will occur at

$$\sin\theta = \frac{h}{2pd} \ .$$

An unscattered electron that was heading toward that position on the screen, meanwhile, would have a y-component of its momentum given by $p_y = p \sin\theta$, or $\sin\theta = p_y / p$. Combining these expressions, we have

$$\sin\theta = \frac{p_y}{p} = \frac{h}{2pd},$$

or

$$p_y = \frac{h}{2d} .$$

In order not to destroy the characteristic interference pattern, we must require that the uncertainty in the momentum after scattering, Δp_y, remains much smaller than the y-component of the unscattered momentum:

$$\Delta p_y << p_y = \frac{h}{2d} .$$

But any definite measurement of which slit the electron passed through also required $\Delta y << d$. Thus in order *both* to measure through which slit the electron passed *and* not to disrupt the characteristic interference pattern, we would require

$$\Delta y \Delta p_y << d\left(\frac{h}{2d}\right) = \frac{h}{2}.$$

Yet this requirement violates Heisenberg's uncertainty principle!

Your assistant helps clarify the situation: the act of measuring through which slit the electron passed destroys the interference pattern. An electron that encounters one of the test particles would necessarily receive such a large "kick," Δp_y, that the minima of the interference pattern would get smeared out, destroying the characteristic pattern of peaks and valleys. In fact, if every electron were definitely measured to pass through either slit *A* or slit *B*, the resulting detection pattern when both slits were open would revert to the sum of the two single-slit distributions—all wavelike behavior would vanish.

This illustrates a more general feature of quantum systems. If you ask a specifically "particle-like" question (such as, "which slit did the particle pass through?"), then you will always get a particle-like response ("slit *A*" or "slit *B*"). If you ask a "wave-like" question (such as "how does the ψ-function behave in the region between the slits and the detectors?"), you will get a wave-like response ("in a state of superposition leading to the characteristic interference pattern"). We need both particle concepts and wave concepts to describe the quantum realm, but now in a rather different combination than we encountered for classical systems.

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Fall 2020

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