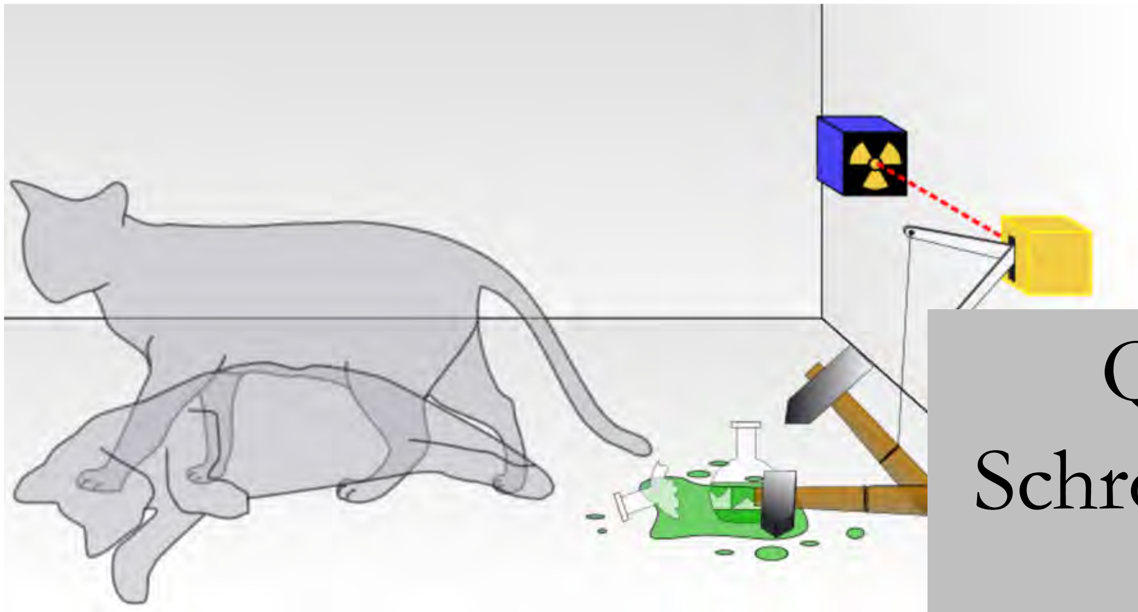


Quantum Weirdness: Schrödinger's Cat, EPR, and Bell's Theorem

8.225 / STS.042, Physics in the 20th Century
Professor David Kaiser, 14 October 2020

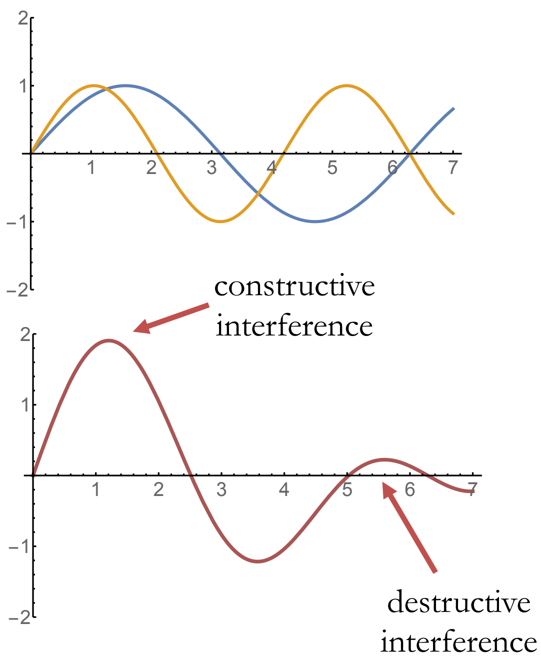
1. Superposition and Schrödinger's Cat

2. EPR and "Elements of Reality"

3. Bell's Inequality and Entanglement*

* See optional *Lecture Notes* on Bell's inequality

Schrödinger's Equation Recap

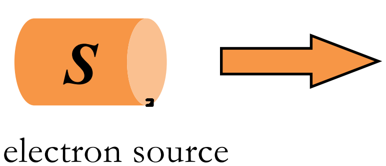


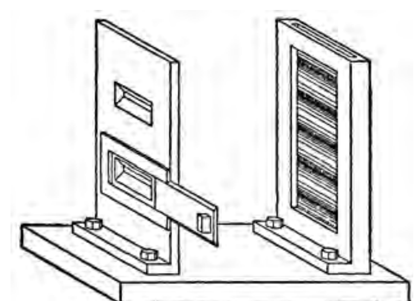
In 1926, *Erwin Schrödinger* developed an approach to a first-principles quantum mechanics, which appeared (at first) to be distinct from Heisenberg's matrix mechanics. Building on *Louis de Broglie's* suggestion about matter waves, Schrödinger introduced a *wave equation* for a new quantity, the *quantum wave function* ψ.

$$E\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V(r)\psi$$

Solutions obeyed *superposition*: if $\psi_1(t,\mathbf{x})$ and $\psi_2(t,\mathbf{x})$ were each solutions, then so was $\psi_3(t,\mathbf{x}) = \psi_1(t,\mathbf{x}) + \psi_2(t,\mathbf{x})$. That yields *interference*.

In summer 1926, *Max Born* suggested that $\psi(t,\mathbf{x})$ is a "probability amplitude."
Probability = $|\psi|^2$.





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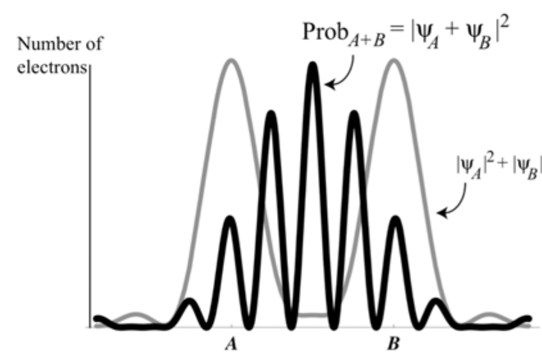
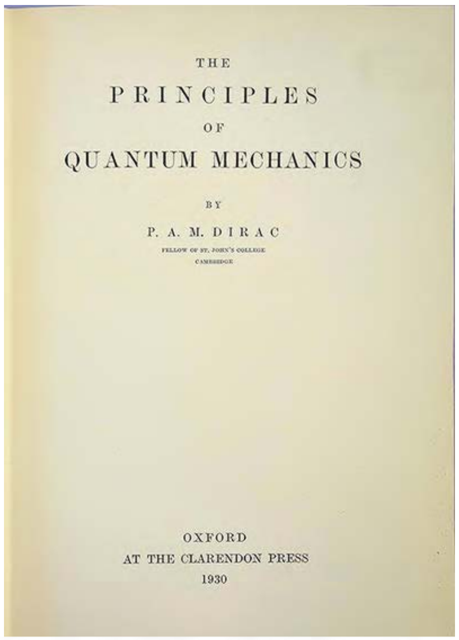


Superposition and Quantum States



British physicist *Paul Dirac* further formalized Schrödinger's approach. In general, a quantum state could be represented as a *vector* $|\psi\rangle$ in an (abstract) vector space. That state vector could itself be represented as a weighted sum of "eigenstates": states with a definite value for a specific property:

$$|\psi\rangle = \sum_n a_n |\phi_n\rangle$$

(This is just a more formal or abstract way of representing *superposition*: if each of the states $|\phi_n\rangle$ is a solution, then so is their sum.)

$$|\psi\rangle = a_{\text{up}} |\uparrow\rangle + a_{\text{down}} |\downarrow\rangle$$

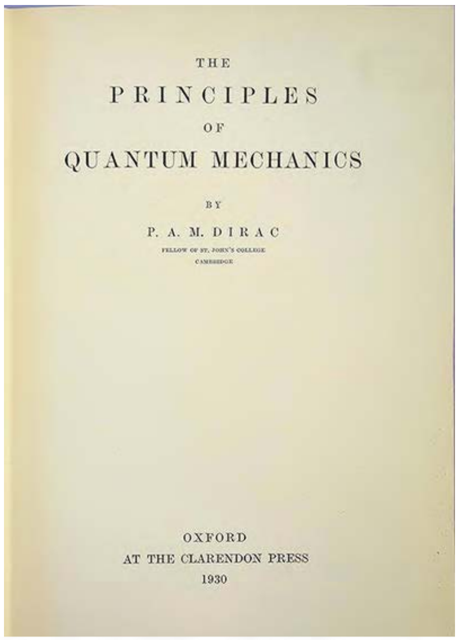
$|\uparrow\rangle$ = spin up along $\hat{\mathbf{z}}$, with $\hat{s}_z |\uparrow\rangle = +\frac{\hbar}{2} |\uparrow\rangle$

$|\downarrow\rangle$ = spin down along $\hat{\mathbf{z}}$, with $\hat{s}_z |\downarrow\rangle = -\frac{\hbar}{2} |\downarrow\rangle$

The two spin states are *orthogonal*; their corresponding state vectors have *vanishing overlap*: $\langle\uparrow|\downarrow\rangle = 0$

Even though the individual eigenstates are incompatible with each other, we may construct a valid quantum state via superposition.

Superposition and Quantum States



$$|\psi\rangle = a_{\rm up}\,|{\uparrow}\rangle + a_{\rm down}\,|{\downarrow}\rangle$$

If we perform a measurement of spin along **z** for a particle prepared in this state:

$$\text{Prob(up)} = \left|\,\langle{\uparrow}|\psi\rangle\,\right|^2 = \left|a_{\rm up}\right|^2$$

$$\text{Prob(down)} = \left|\,\langle{\downarrow}|\psi\rangle\,\right|^2 = \left|a_{\rm down}\right|^2$$

Each time the spin is measured along **z**, we always find a *definite* result: either spin up or spin down. We never find a "smeared out" or fuzzy result.

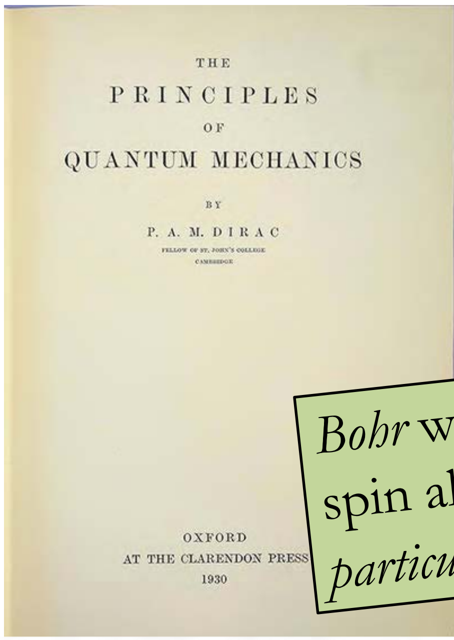
$$|{\uparrow}\rangle \;= \text{spin up along } \hat{\mathbf{z}}, \qquad \text{with } \hat{s}_z\,|{\uparrow}\rangle = +\tfrac{\hbar}{2}\,|{\uparrow}\rangle$$

$$|{\downarrow}\rangle \;= \text{spin down along } \hat{\mathbf{z}}, \qquad \text{with } \hat{s}_z\,|{\downarrow}\rangle = -\tfrac{\hbar}{2}\,|{\downarrow}\rangle$$

The two spin states are *orthogonal*; their corresponding state vectors have *vanishing overlap*: $\langle{\uparrow}|{\downarrow}\rangle = 0$

And yet, according to *Born*, *Dirac*, *Bohr* and the others, quantum mechanics can only be used to calculate *probabilities*; the equations *offer no way to know in advance* whether the particle was *really* spin up or spin down, prior to the measurement.

Superposition and Quantum States



$$|\psi\rangle = a_{\rm up}\,|\uparrow\rangle + a_{\rm down}\,|\downarrow\rangle$$

If we perform a measurement of spin along **z** for a particle prepared in this state:

$$\text{Prob(up)} = |\langle\uparrow|\psi\rangle|^2 \ldots$$

Bohr went further: what if the particle *had no definite value* (e.g., of spin along **z**) prior to its measurement? As if a person had *no particular weight* until stepping on a bathroom scale.

… a *definite* result:

… n. We never find a "smeared out" or fuzzy result.

$|\uparrow\rangle$ = spin up along $\hat{\mathbf{z}}$, with $\hat{s}_z\,|\uparrow\rangle = +\frac{\hbar}{2}\,|\uparrow\rangle$

$|\downarrow\rangle$ = spin down along $\hat{\mathbf{z}}$, with $\hat{s}_z\,|\downarrow\rangle = -\frac{\hbar}{2}\,|\downarrow\rangle$

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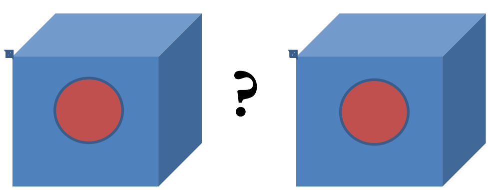
And yet, according to *Born*, *Dirac*, *Bohr* and the others, quantum mechanics can only be used to calculate *probabilities*; the equations *offer no way to know in advance* whether the particle was *really* spin up or spin down, prior to the measurement.

Schrödinger's Cat

Einstein grew frustrated with this restriction to only calculating *probabilities*. He wrote to his good friend *Max Born* in December 1926:

"Quantum mechanics is certainly imposing. But an inner voice tells me it is not yet the real thing. The theory says a lot, but does not really bring us any closer to the secret of the 'old one.' I, at any rate, am convinced that *He* is not playing at dice."

Einstein and Schrödinger began discussing these points together as well, once Schrödinger moved to Berlin in 1927. (He succeeded *Max Planck*.) They enjoyed *Wiener Würstelabende* evenings together (Viennese sausage parties), and sailing on the lake near Einstein's summer home.



In one letter, Einstein asked Schrödinger to imagine that a ball had been placed in one of two identical, closed boxes. Prior to opening either box, the probability of finding the ball in Box 1 was 50%. "Is this a complete description? *NO:* A complete statement is: the ball *is* (or is not) in the first box."

Schrödinger's Cat

Nazi book-burning rally in Berlin, spring 1933
Image is in the public domain.

Both Einstein and Schrödinger left Germany once the Nazis took power in 1933. They continued their discussions by letter; their examples began to reflect the darker times.

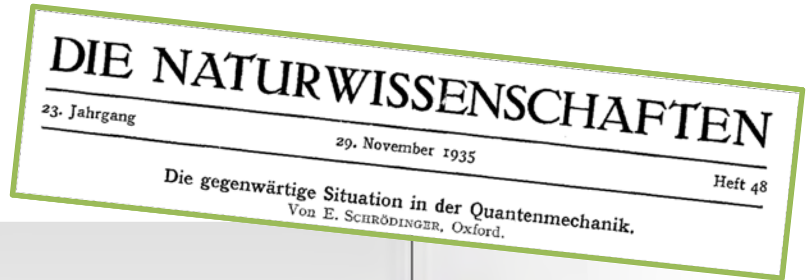
Einstein to *Schrödinger*, 8 August 1935: Imagine a charge of gunpowder that was intrinsically unstable, with 50-50 odds of exploding over the course of one year. "In principle this can quite easily be represented quantum-mechanically":

$$|\psi\rangle_{\text{gunpowder}} = \frac{1}{\sqrt{2}}\left[|\psi\rangle_{\text{unexploded}} + |\psi\rangle_{\text{exploded}}\right]$$

"After the course of a year this is no longer the case at all. Rather, the ψ-function then describes a sort of blend of not-yet and of already-exploded systems. [... But] in reality there is *just no intermediary* between exploded and not-exploded."

Schrödinger's Cat

Schrödinger replied to *Einstein* on 19 August 1935, with a twist:

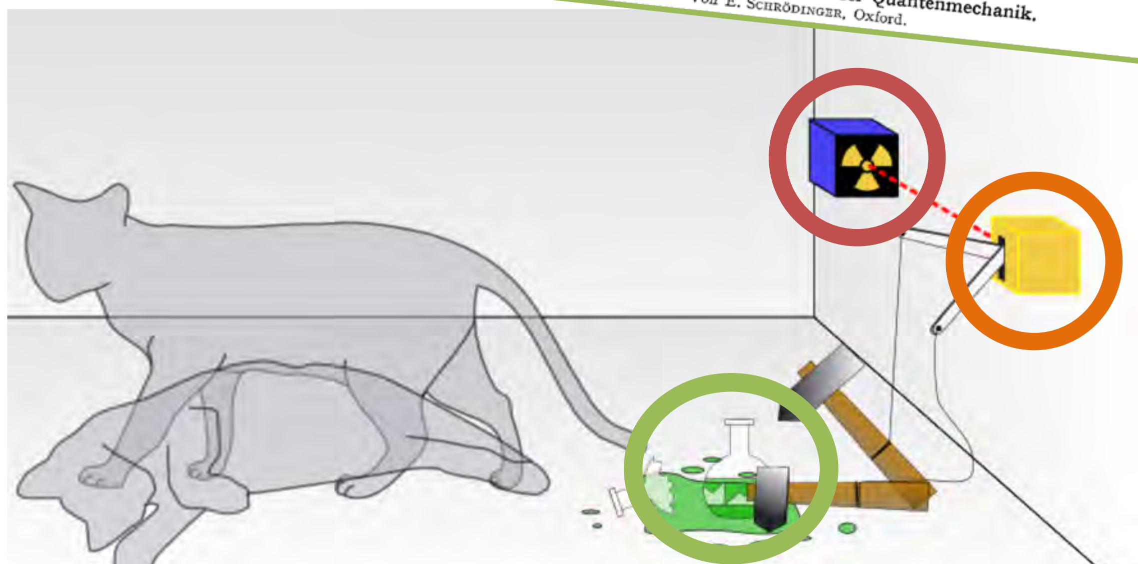

DIE NATURWISSENSCHAFTEN

23. Jahrgang — 29. November 1935 — Heft 48

Die gegenwärtige Situation in der Quantenmechanik.

Von E. Schrödinger, Oxford.

"Confined in a steel chamber is a **Geiger counter** prepared with a tiny amount of [radioactive] uranium, so small that in the next hour it is just as probable to expect one atomic decay as none. An **amplified relay** provides that the first atomic decay shatters a small **bottle of prussic acid** [cyanide poison]. This and — cruelly — a cat is also trapped in the steel chamber."



"After one hour, the living and dead cat are smeared out in equal measure."

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left\{|\psi_{\text{living}}\rangle + |\psi_{\text{dead}}\rangle\right\}$$

Schrödinger's Cat

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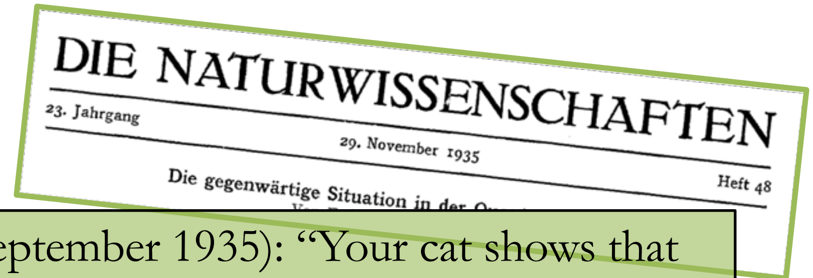
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Einstein's reply (4 September 1935): "Your cat shows that we are in complete agreement. A ψ-function that contains the living as well as the dead cat just cannot be taken as a description of the real state of affairs."

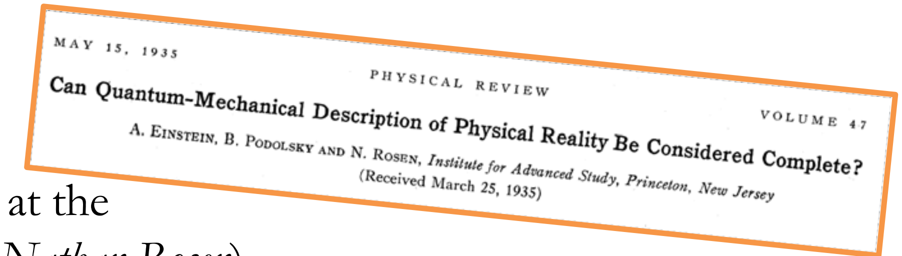
An irony: By early mid-1930s, Schrödinger had become *skeptical* of some of his own contributions to quantum mechanics, and — along with Einstein — became an outspoken critic. He invented his cat paradox as a *critique* of the central role of superposition and probabilities in quantum theory.

"After one hour, the living and dead cat are smeared out in equal measure."

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left\{|\psi_{\text{living}}\rangle + |\psi_{\text{dead}}\rangle\right\}$$

Questions?

EPR and "Elements of Reality"

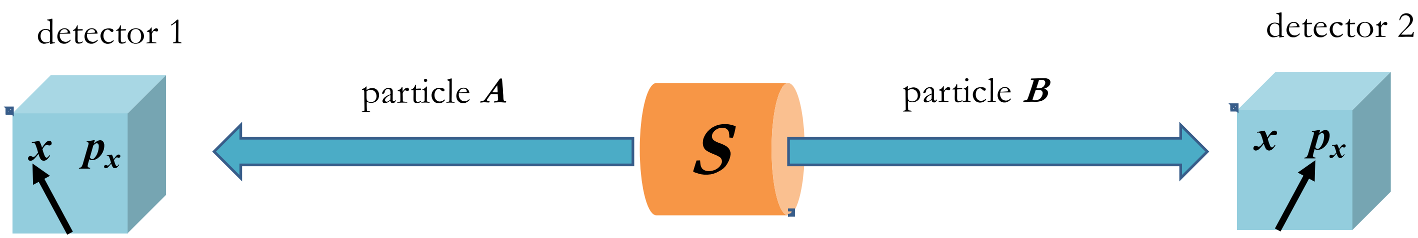

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

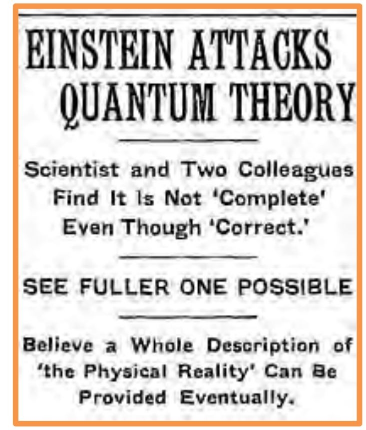
Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

That same year (1935), *Einstein* and two younger colleagues at the Institute for Advanced Study in Princeton (*Boris Podolsky* and *Nathan Rosen*) published an even more elaborate critique of quantum theory.




EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

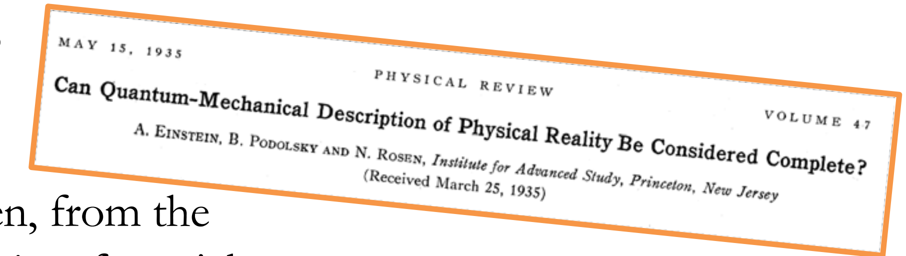
Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

According to quantum mechanics, prior to either measurement the system would be described by a *superposition* of two different two-particle states:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left\{ |u\rangle_A |v\rangle_B + |v\rangle_A |u\rangle_B \right\}$$

The quantum state does not *factorize*: according to quantum mechanics, there is no way to describe the behavior of particle ***A*** without referring to the behavior of particle ***B***.

EPR and "Elements of Reality"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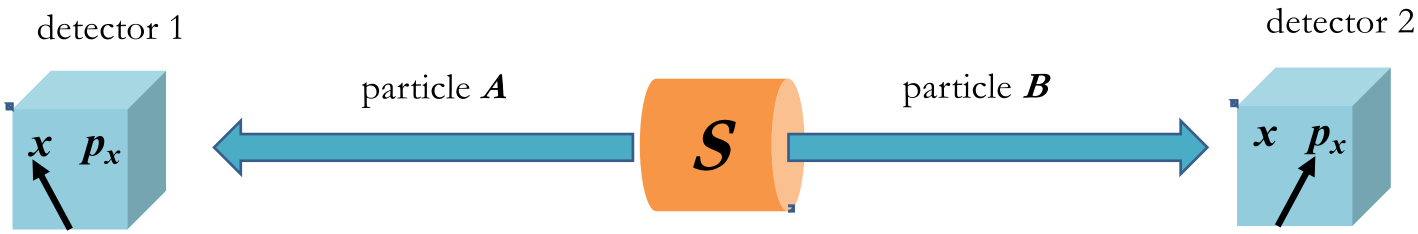
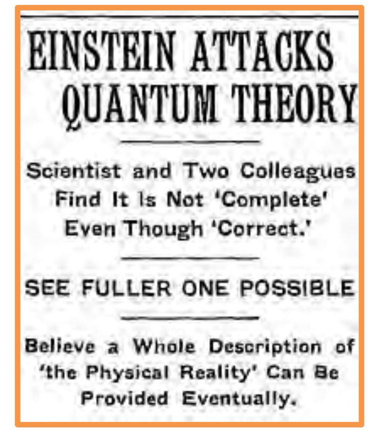
Suppose physicist 1 measures the *position* of particle ***A***. Then, from the conservation of momentum, she immediately knows the *position* of particle ***B***. *Or* physicist 1 could choose to measure the *momentum* of particle ***A***.

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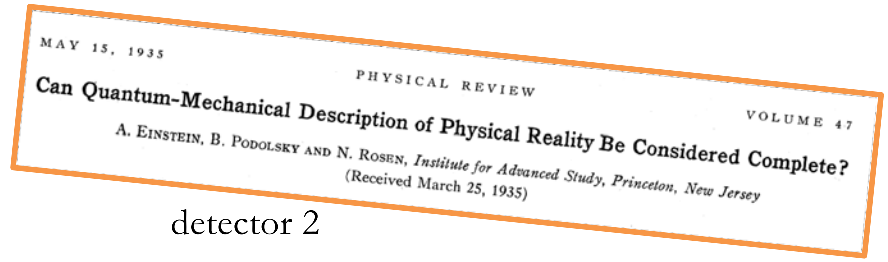
If she waited until the last possible moment to decide which measurement to perform, there would not be enough time for a signal to *update* particle ***B*** as to what values it should have for various properties. So particle ***B*** must have carried its own set of definite properties *on its own*, prior to measurement.

Quantum mechanics does not describe particle ***B***'s properties on its own. Therefore *quantum mechanics must be incomplete.*

EPR and "Elements of Reality"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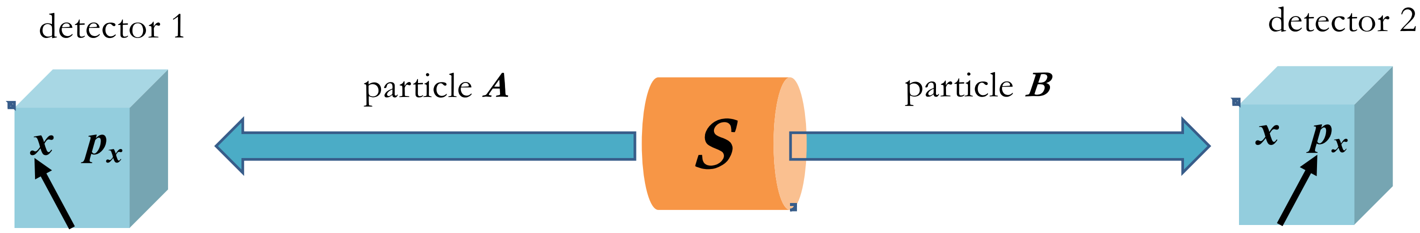
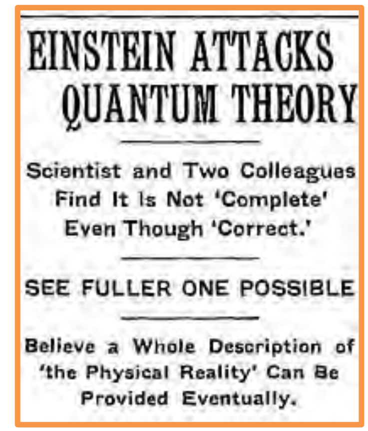
The EPR conclusion relied upon *two assumptions*:

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"**Reality criterion**" (p. 777): "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of reality corresponding to this physical quantity."

This is an assumption that quantum objects possess *complete sets of properties on their own*, prior to our efforts to measure them.

"**Locality**" (p. 779): "Since at the time of measurement the two systems [particles ***A*** and ***B***] no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system."

This is an assumption that no influence can travel *arbitrarily quickly*: local causes yield local effects.

EPR and “Elements of Reality”

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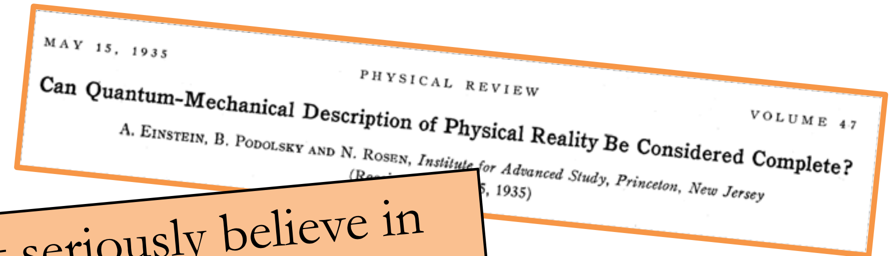
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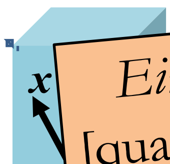
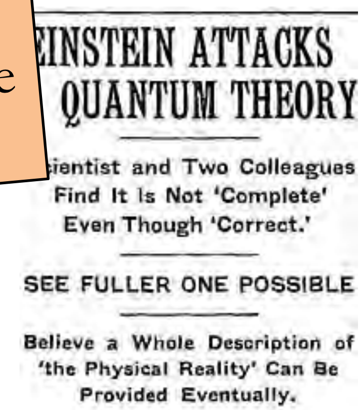
(Re… …, 1935)

detector 1



Einstein to *Max Born*, 3 March 1947: “I cannot seriously believe in [quantum mechanics] because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from ***spooky actions at a distance***.”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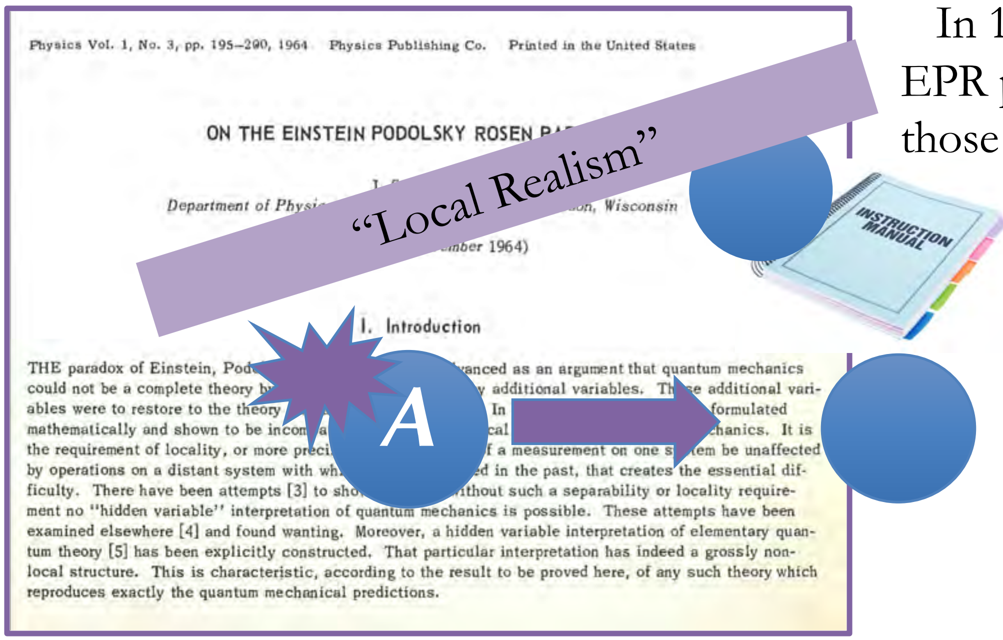
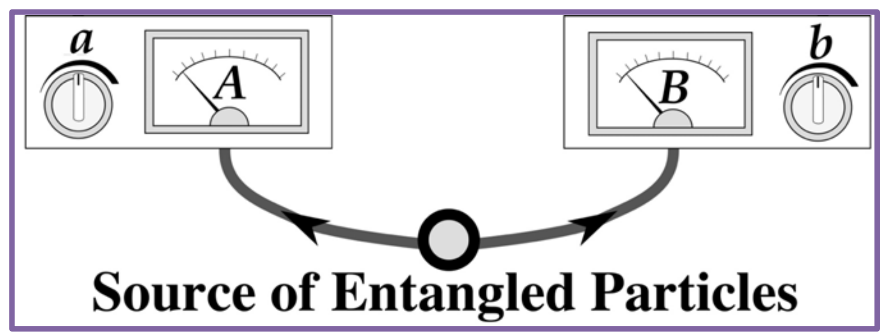
Questions?

Bell's Inequality and Entanglement



In 1964, Irish physicist *John S. Bell* scrutinized the EPR paper, returning to the two main assumptions that those authors had made:

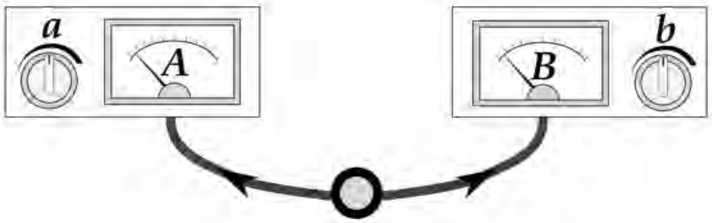
- Each particle has definite properties, on its own, before it is measured. (**Reality criterion**)
- No influence can travel across space arbitrarily quickly. (**Locality**)



detector settings: a, b
measurement outcomes: A, B

Bell suggested: these are *assumptions*, and we can try to *test* them.

Correlations at a Distance



Dichotomic observables $A(a), B(b) = \pm 1$

Correlation functions $E(a,b) = \langle A(a)B(b) \rangle$

$$S = E(a,b) + E(a',b) - E(a,b') + E(a',b')$$

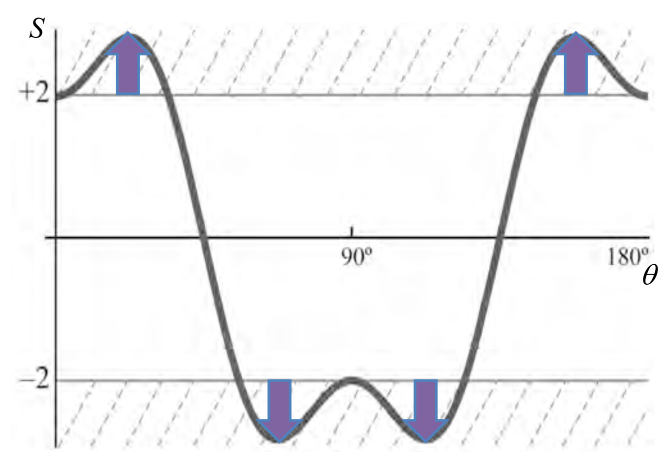
Bell: if $p(A,B|a,b) = \int d\lambda\, p(\lambda)\, \underbrace{p(A|a,\lambda)}\, p(B|b,\lambda)$

then $|S| \leq 2.$

(**Locality**: A does not depend on b or B, and vice versa.)

Bell's inequality

QM prediction:
$S_{max} = 2\sqrt{2}$



Quantum mechanics predicts that quantum systems can be *more strongly correlated* than Bell's inequality would allow.


Alice
Bob
Cambridge, MA
Cambridge, UK

Alice

Bob

Cambridge, MA

Cambridge, UK

“What would you like for dessert?”

Alice

Bob

Cambridge, MA

Cambridge, UK

"What would you like for dessert?"

or

Alice

Bob

Cambridge, MA

Cambridge, UK

"What would you like for dessert?"

a  +1 *or*  -1

Cambridge, MA

Cambridge, UK

"What would you like for dessert?"

a

+1

or

-1

a′

+1

or

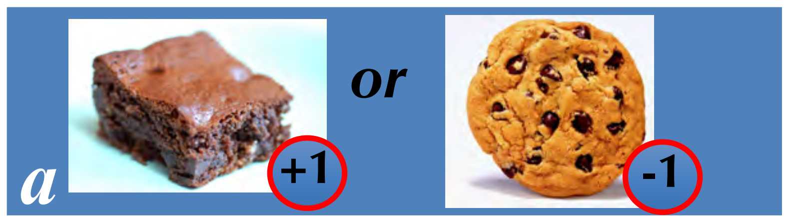
-1

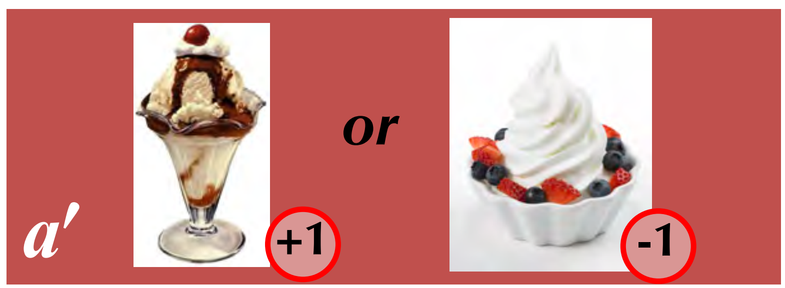

Alice
Bob
Cambridge, MA
Cambridge, UK
"What would you like for dessert?"
a
or
+1
-1
a′
or
+1
-1

Cambridge, MA

Cambridge, UK

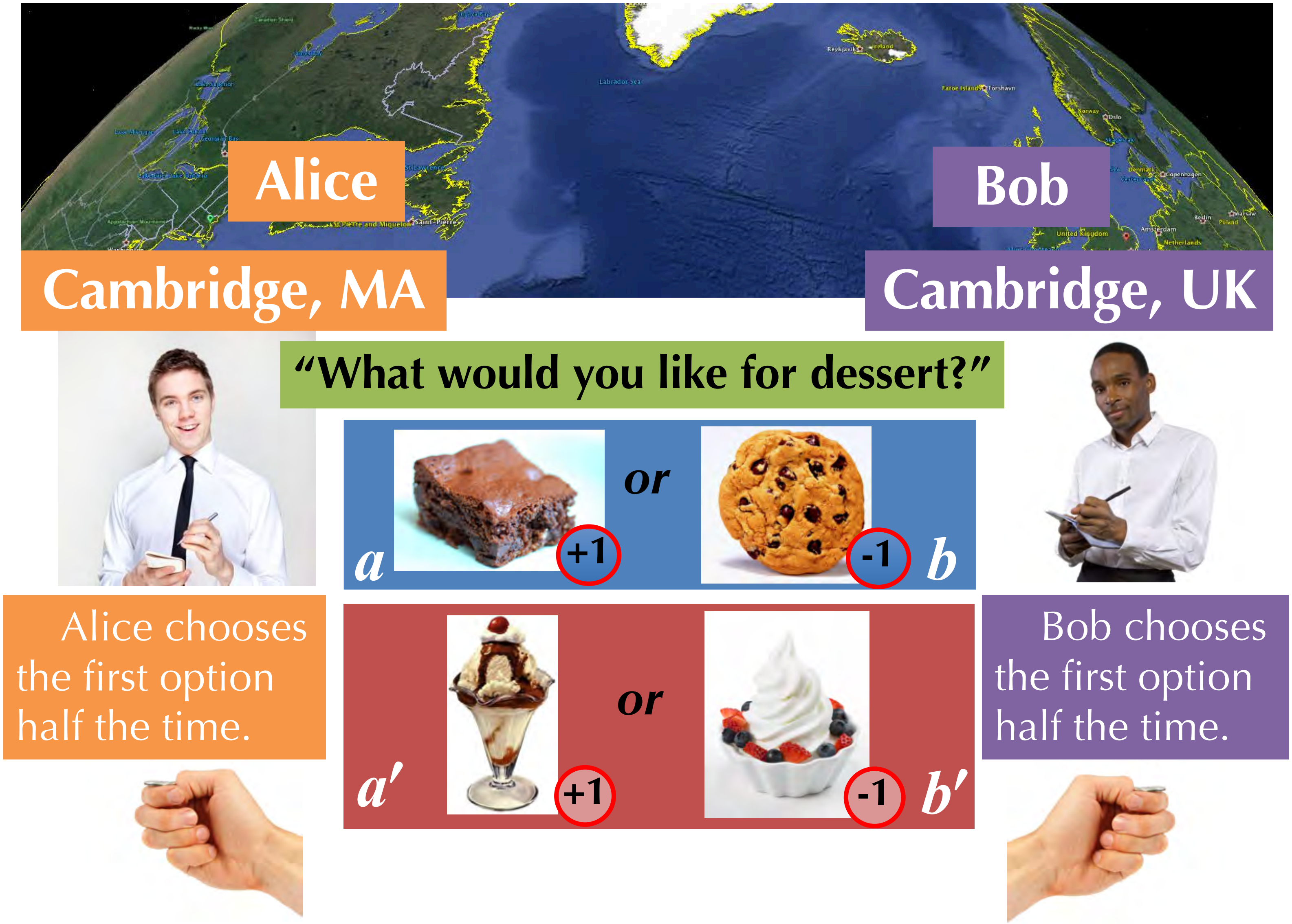
"What would you like for dessert?"





Alice chooses the first option half the time.


Alice
Bob
Cambridge, MA
Cambridge, UK
"What would you like for dessert?"
a
+1
or
-1
b
a′
+1
or
-1
b′
Alice chooses the first option half the time.


Alice
Bob
Cambridge, MA
Cambridge, UK
"What would you like for dessert?"
a
+1
or
-1
b
a′
+1
or
-1
b′
Alice chooses the first option half the time.
Bob chooses the first option half the time.

Alice

Bob

Cambridge,

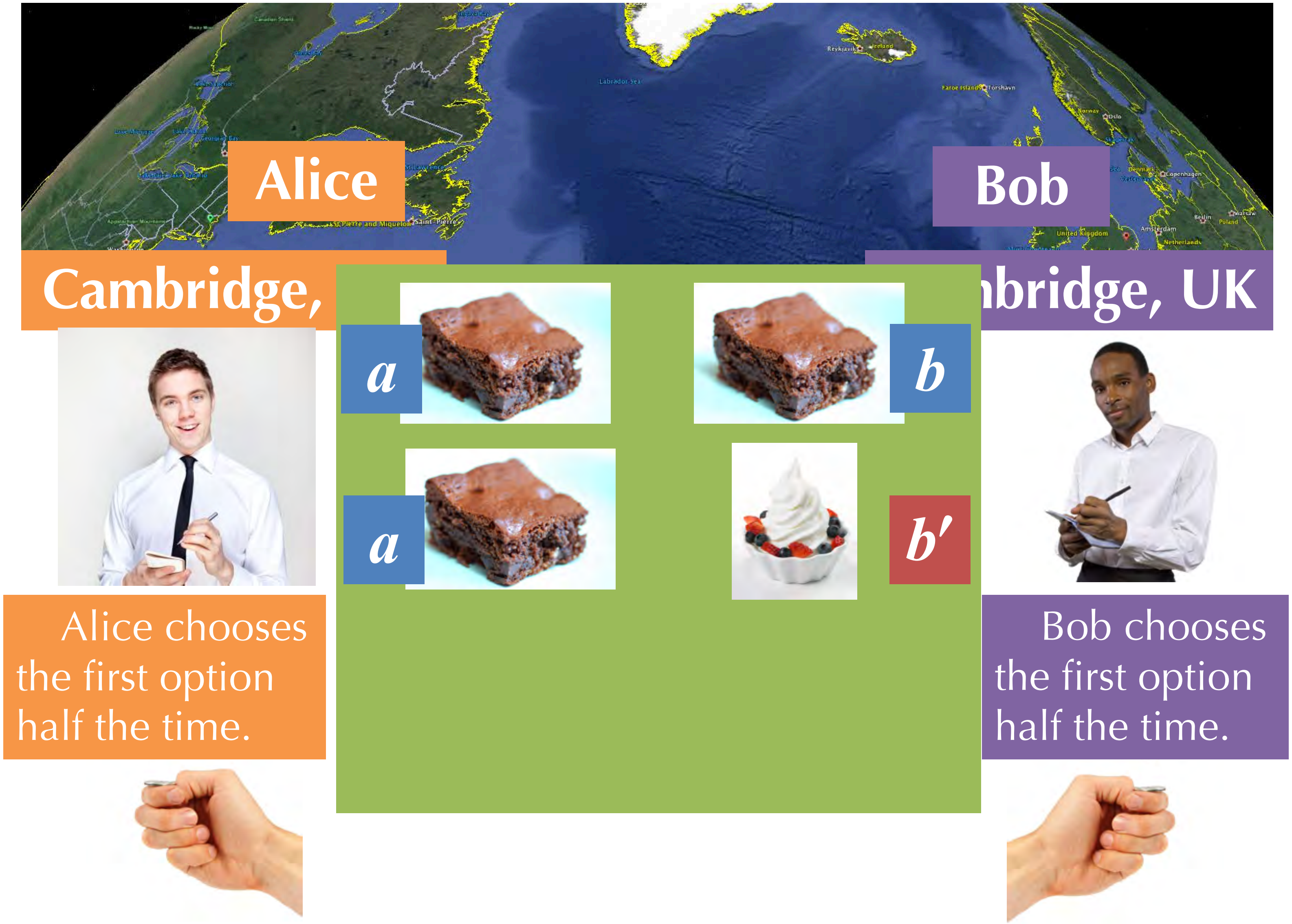
bridge, UK

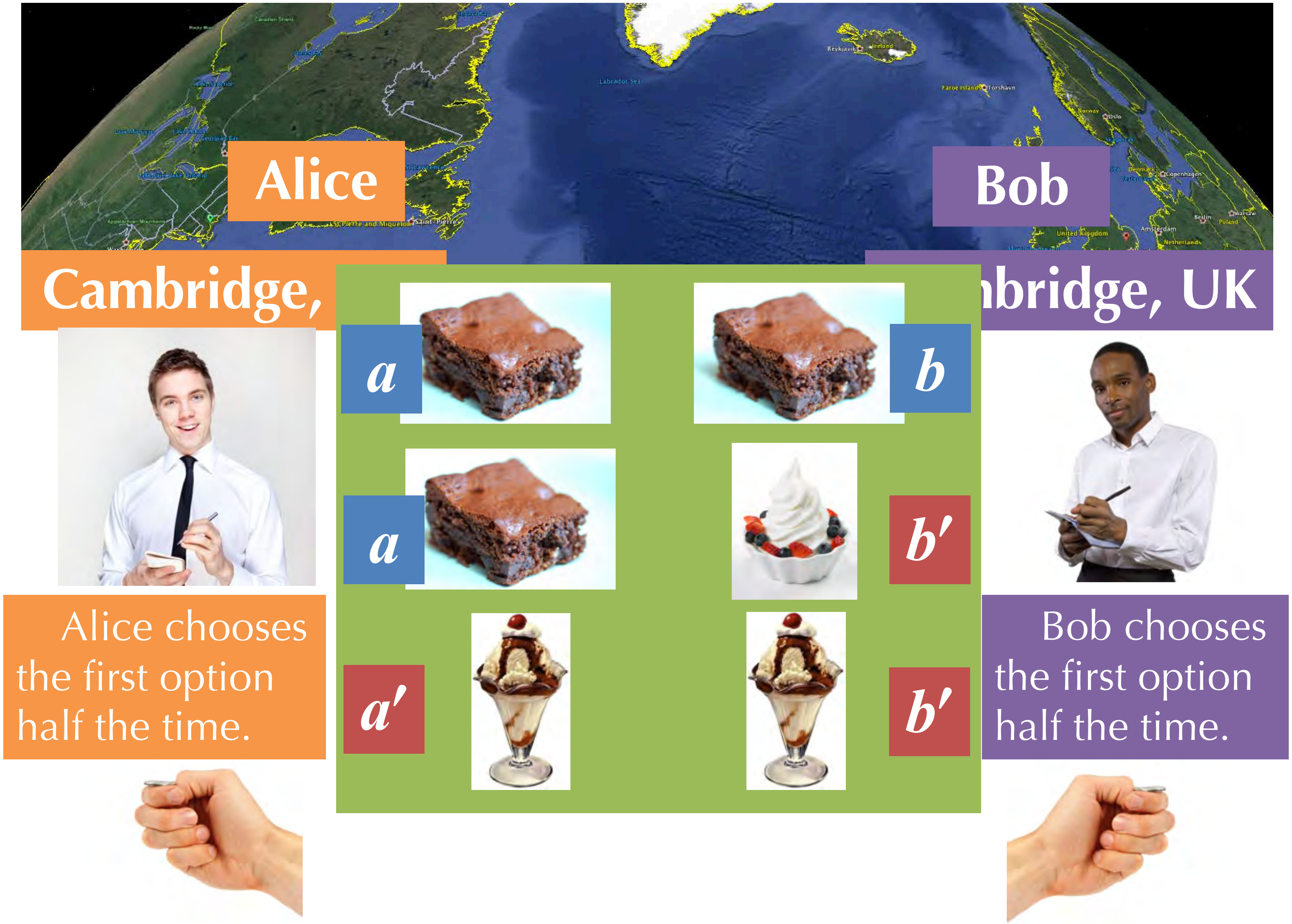
a

b

Alice chooses the first option half the time.

Bob chooses the first option half the time.


Alice
Bob
Cambridge,
bridge, UK
a
b
a
b′
Alice chooses the first option half the time.
Bob chooses the first option half the time.


Alice
Bob
Cambridge,
bridge, UK
a
b
a
b′
a′
b′
Alice chooses the first option half the time.
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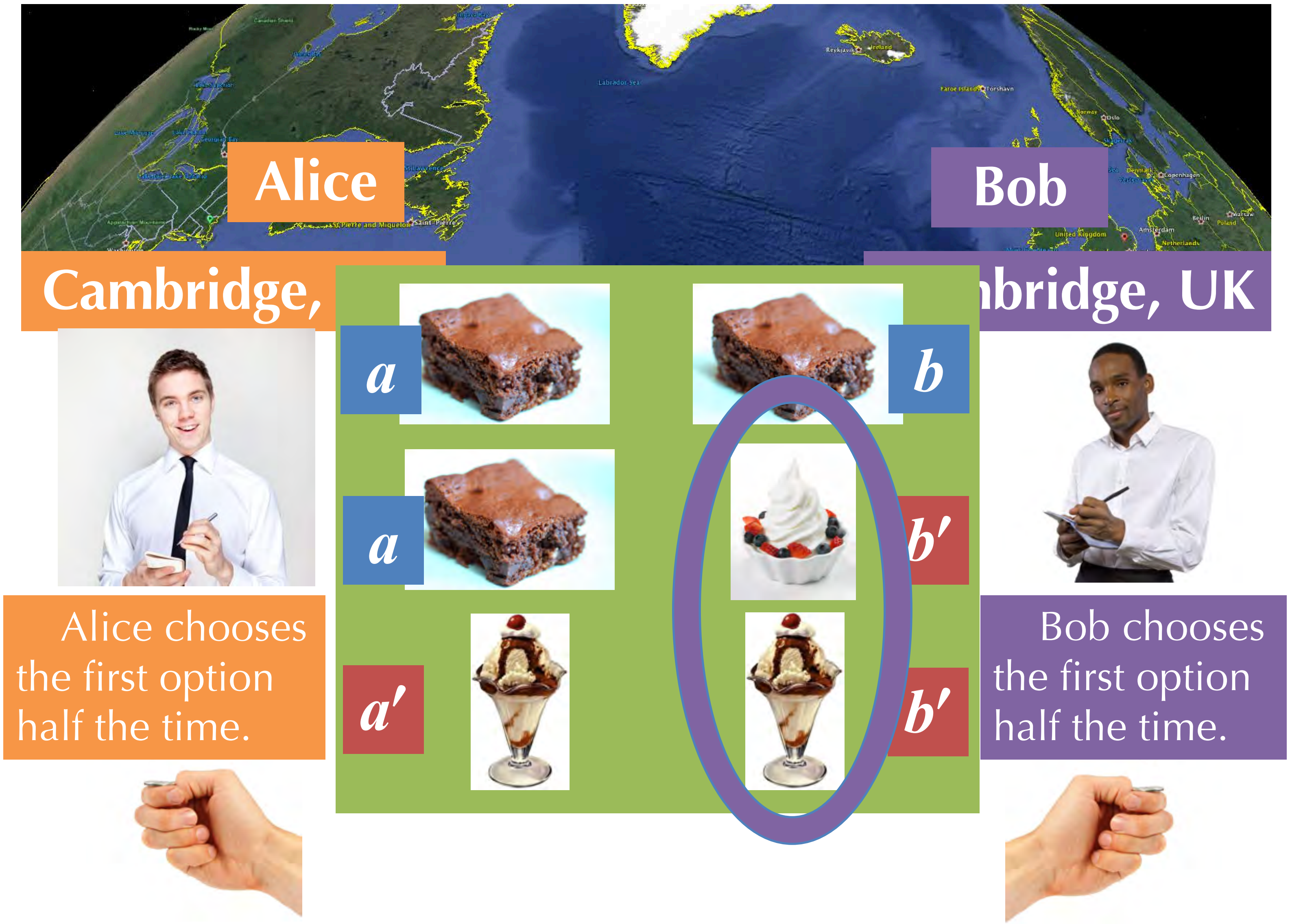
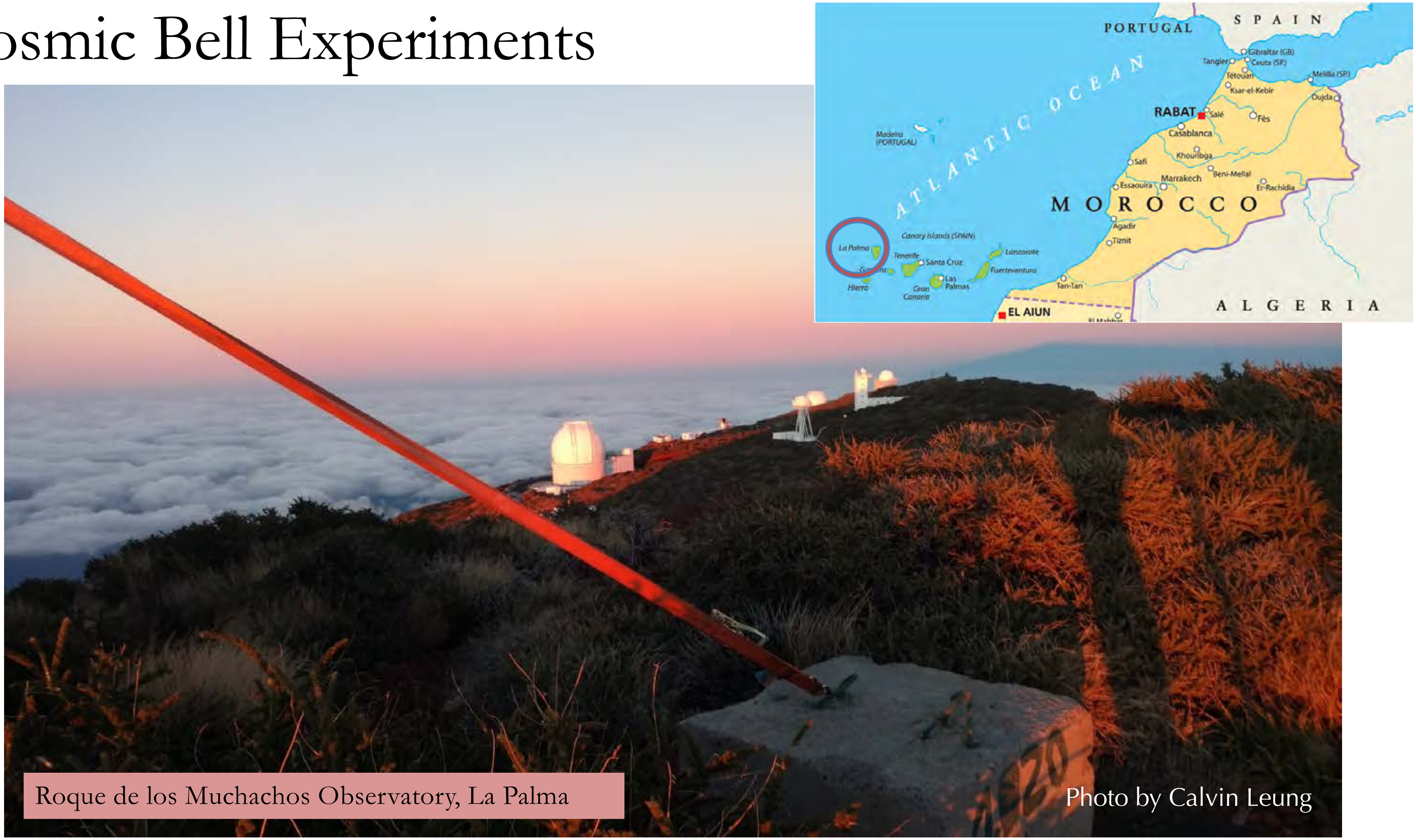

Alice
Bob
Cambridge,
bridge, UK
a
a
a′
b
b′
b′
Alice chooses the first option half the time.
Bob chooses the first option half the time.

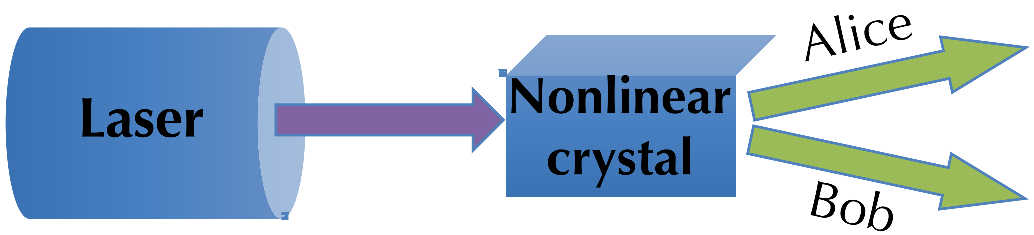

"Spooky" correlations!
Alice
Bob
Cambridge,
nbridge, UK
a
a
a′
b
b′
b′
Alice chooses the first option half the time.
Bob chooses the first option half the
B
?!

Cosmic Bell Experiments



Roque de los Muchachos Observatory, La Palma

Photo by Calvin Leung



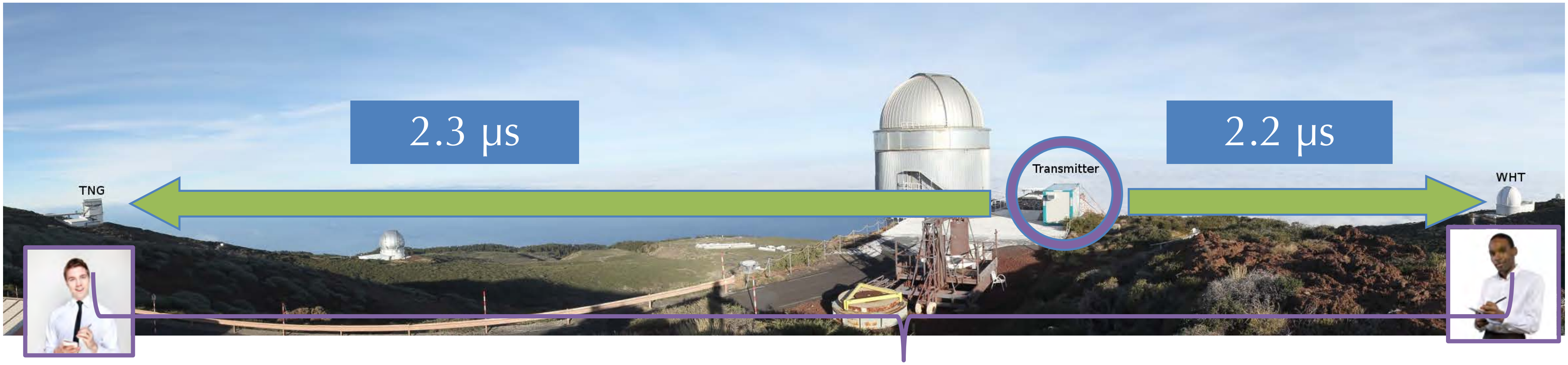

PHYSICAL REVIEW LETTERS **121,** 080403 (2018)

Editors' Suggestion

Cosmic Bell Test Using Random Measurement Settings from High-Redshift Quasars

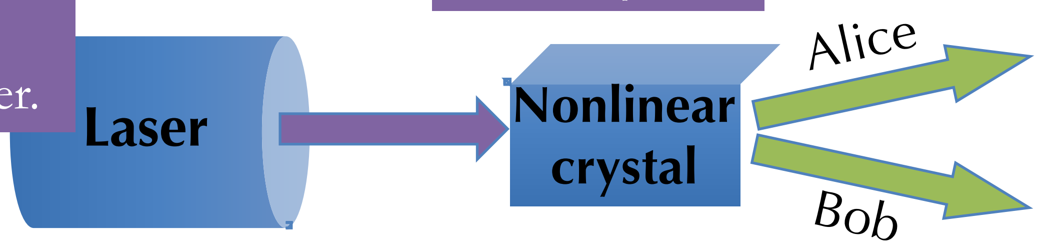
Dominik Rauch,[1,2,*] Johannes Handsteiner,[1,2] Armin Hochrainer,[1,2] Jason Gallicchio,[3] Andrew S. Friedman,[4] Calvin Leung,[1,2,3,5] Bo Liu,[6] Lukas Bulla,[1,2] Sebastian Ecker,[1,2] Fabian Steinlechner,[1,2] Rupert Ursin,[1,2] Beili Hu,[3] David Leon,[4] Chris Benn,[7] Adriano Ghedina,[8] Massimo Cecconi,[8] Alan H. Guth,[5] David I. Kaiser,[5,†] Thomas Scheidl,[1,2] and Anton Zeilinger[1,2,‡]


arXiv:1808.05966



3.4 μs

Not enough time for a single light beam to have traveled from one detector to the other.

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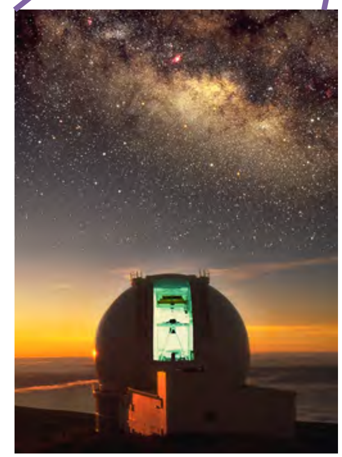
Galileo National Telescope

William Herschel Telescope



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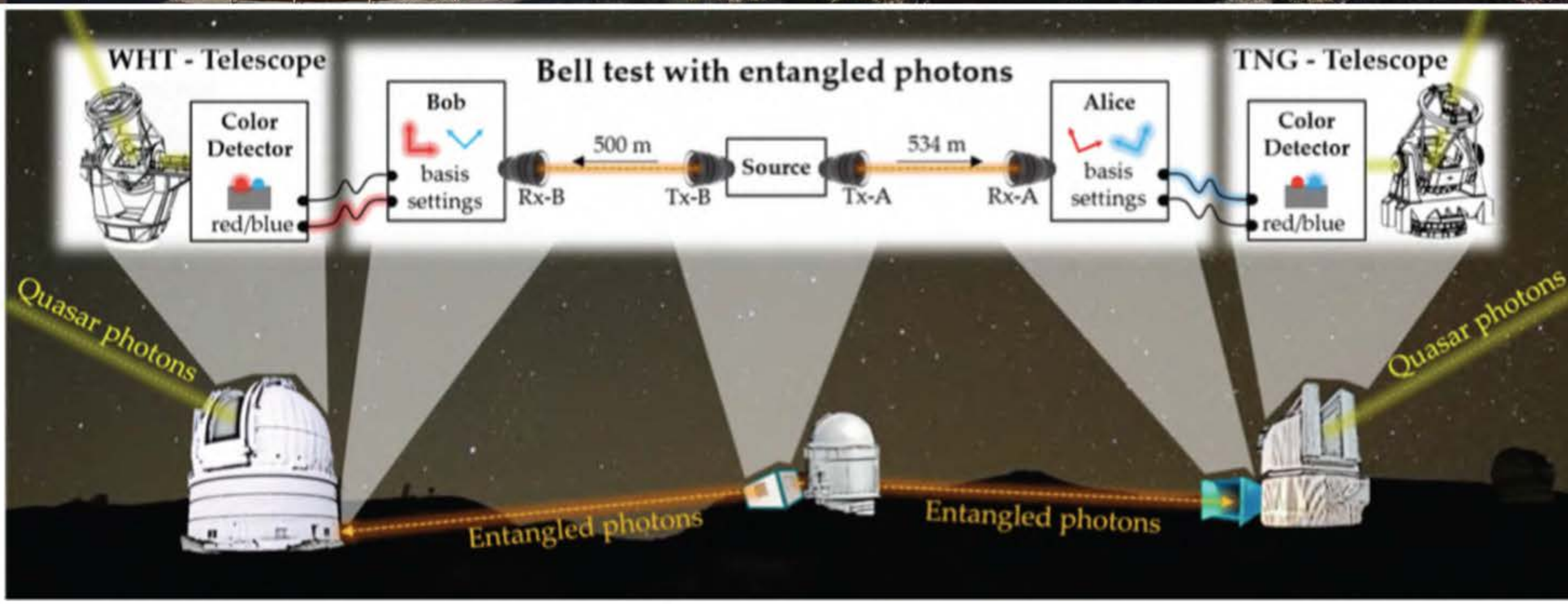
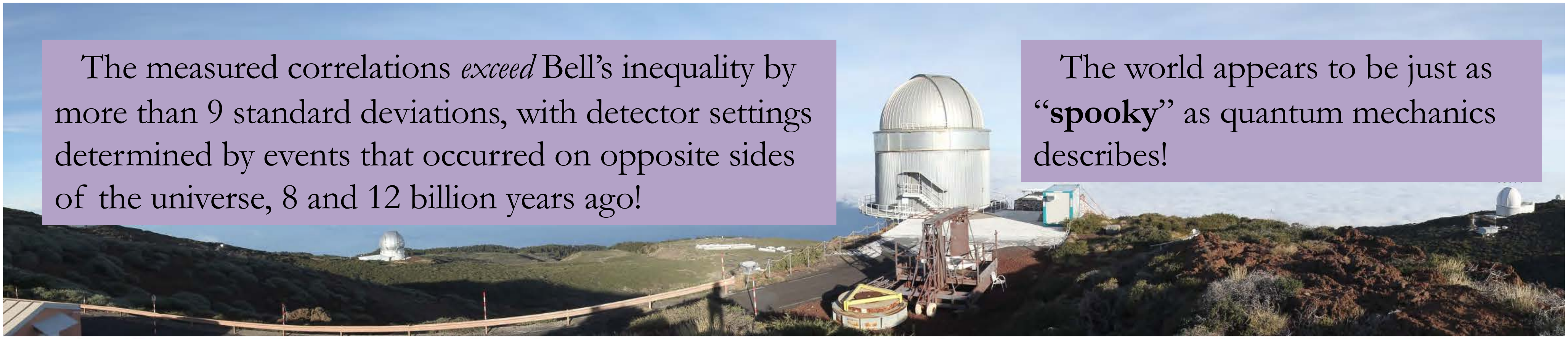
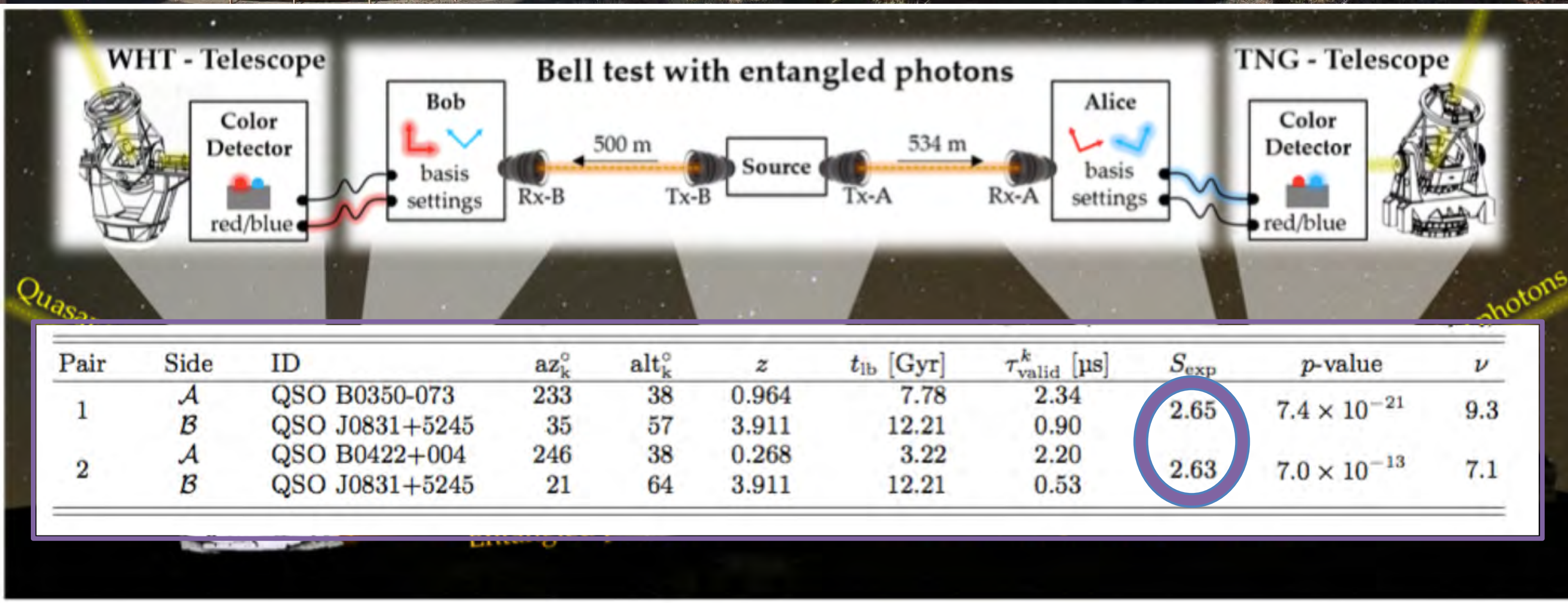
TNG
Transmitter
WHT


WHT - Telescope
Bell test with entangled photons
TNG - Telescope
Color Detector
red/blue
Bob
basis settings
500 m
Source
534 m
Alice
basis settings
Rx-B
Tx-B
Tx-A
Rx-A
Color Detector
red/blue
Quasar photons
Quasar photons
Entangled photons
Entangled photons

The measured correlations *exceed* Bell's inequality by more than 9 standard deviations, with detector settings determined by events that occurred on opposite sides of the universe, 8 and 12 billion years ago!

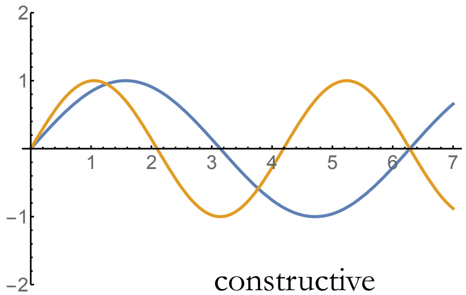
The world appears to be just as "**spooky**" as quantum mechanics describes!

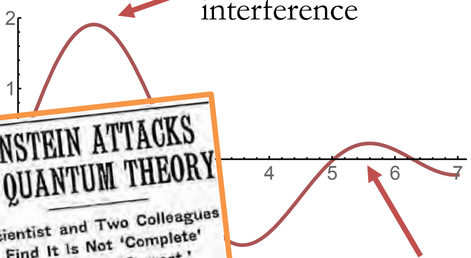


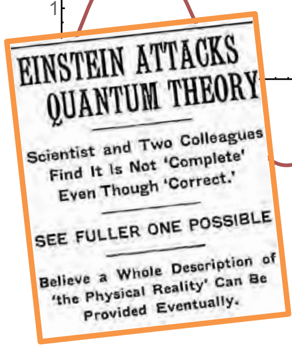


Pair	Side	ID	az°_{k}	alt°_{k}	z	t_{lb} [Gyr]	τ^{k}_{valid} [μs]	S_{exp}	p-value	ν
1	$\mathcal{A}$	QSO B0350-073	233	38	0.964	7.78	2.34	2.65	7.4×10^{-21}	9.3
	$\mathcal{B}$	QSO J0831+5245	35	57	3.911	12.21	0.90			
2	$\mathcal{A}$	QSO B0422+004	246	38	0.268	3.22	2.20	2.63	7.0×10^{-13}	7.1
	$\mathcal{B}$	QSO J0831+5245	21	64	3.911	12.21	0.53			

Quantum Weirdness Summary







EINSTEIN ATTACKS QUANTUM THEORY

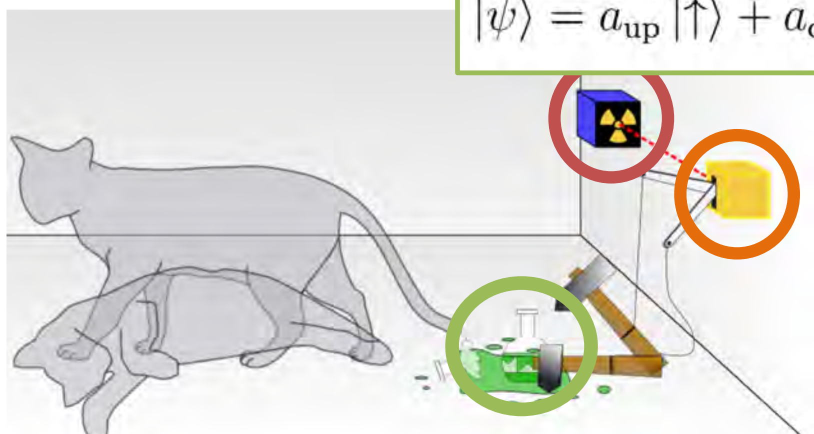
Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

Even though the individual eigenstates are incompatible with each other, we may construct a valid quantum state via *superposition*:

$$|\psi\rangle = a_{\text{up}} |\uparrow\rangle + a_{\text{down}} |\downarrow\rangle$$



$$|\psi\rangle = \frac{1}{\sqrt{2}}\left\{|\psi_{\text{living}}\rangle + |\psi_{\text{dead}}\rangle\right\}$$

Are *probabilities* calculated from *superpositions* really enough to describe our world?

Can we make sense of the behavior of *entangled* systems, which do not obey “local realism”?