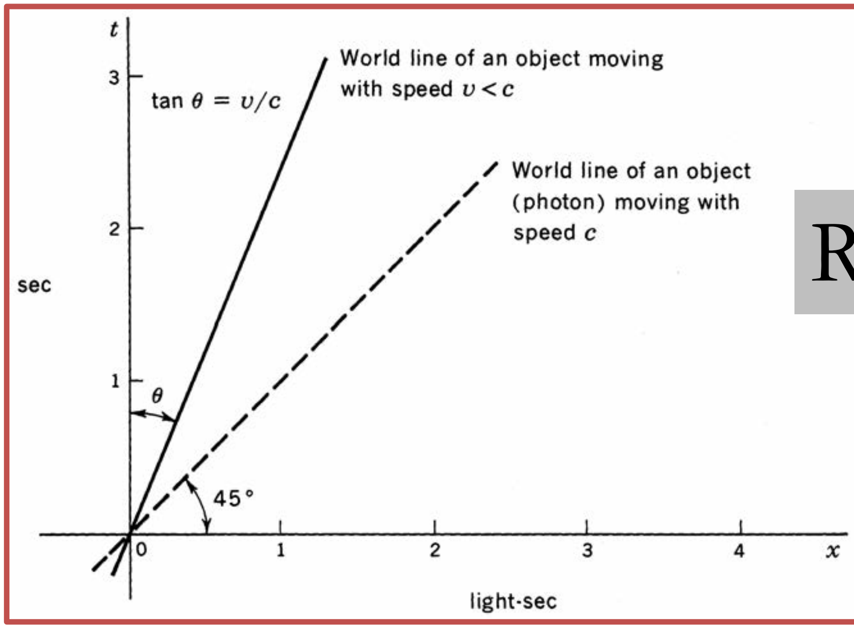


Reception of Special Relativity

8.225 / STS.042, Physics in the 20th Century
Professor David Kaiser, 23 September 2020

1. Early Reactions to Relativity

2. Minkowski and Spacetime

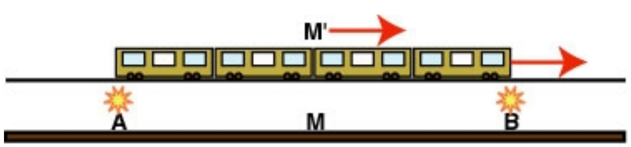
3. Cambridge Wranglers and Relativity

Relativity Recap

"Olympia Academy": Solovine, Habicht, Einstein, ca. 1905

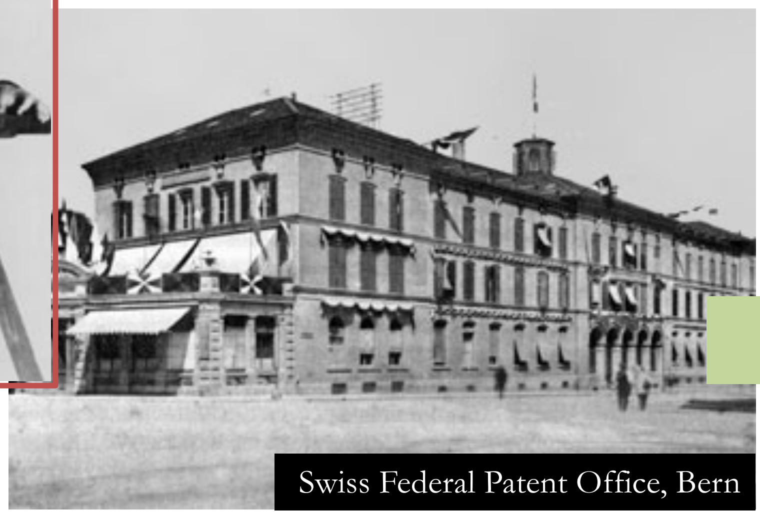
Einstein was inspired by Ernst Mach's "positivism" to focus on "objects of positive experience" — phenomena that could be observed or measured — and hence began by focusing on *kinematics* (motion of objects) rather than *dynamics* (study of forces).

1. The laws of physics are valid in any frame of reference moving at a constant speed (*inertial* frames of reference).
2. The speed of light *c* is constant, independent of the motion of the source.



From the *relativity of simultaneity*, Einstein was led to consider *length contraction* and *time dilation*: all based on how (moving) observers would perform measurements, rather than about forces in the ether.

Early Reactions to Relativity



Swiss Federal Patent Office, Bern

Second reaction: Einstein's 1905 paper presented a clever *re-derivation* of results that had previously been found by physicists like Hendrik Lorentz in the 1890s. After all, Lorentz had already published on *length contraction* and derived the factor $\gamma = 1/[1 - (v/c)^2]^{1/2}$.

"Lorentz-Einstein theory"

Few recognized that Lorentz's work *assumed* an ether, whereas Einstein had dismissed the ether as "superfluous."

As late as 1913, a leading British physicist wrote that the "abstruse conceptions" of Einstein's work were "most foreign to our habits of thought," and "as yet scarcely anyone in this country professed to understand, or at least to appreciate them."

Early Reactions to Relativity

Swiss Federal Patent Office, Bern

"Lorentz-Einstein theory"

1909: assistant professor in Zurich
1911: professor in Prague
1912: professor in Zurich
1914: Prussian Academy of Sciences, Berlin

Questions?

Minkowski and Spacetime

ETH Zürich, ca. 1900.

One of the few researchers who *did* begin to pay attention to Einstein's work was *Herman Minkowski*, one of Einstein's former mathematics teachers at the ETH* in Zürich.

Einstein used to cut Minkowski's classes as a student — borrowing notes from friends to cram before the exams — and, not surprisingly, Minkowski thought little of Einstein's talents. After a friend of Einstein's encouraged Minkowski to read Einstein's 1905 paper, Minkowski replied: "I really wouldn't have thought Einstein capable of that."

To another, Minkowski recalled, "It came as a tremendous surprise, for in his student days Einstein had been a lazy dog. ... He never bothered about mathematics at all."

*Eidgenössische Technische Hochschule: Federal Polytechnic Institute

Minkowski and Spacetime

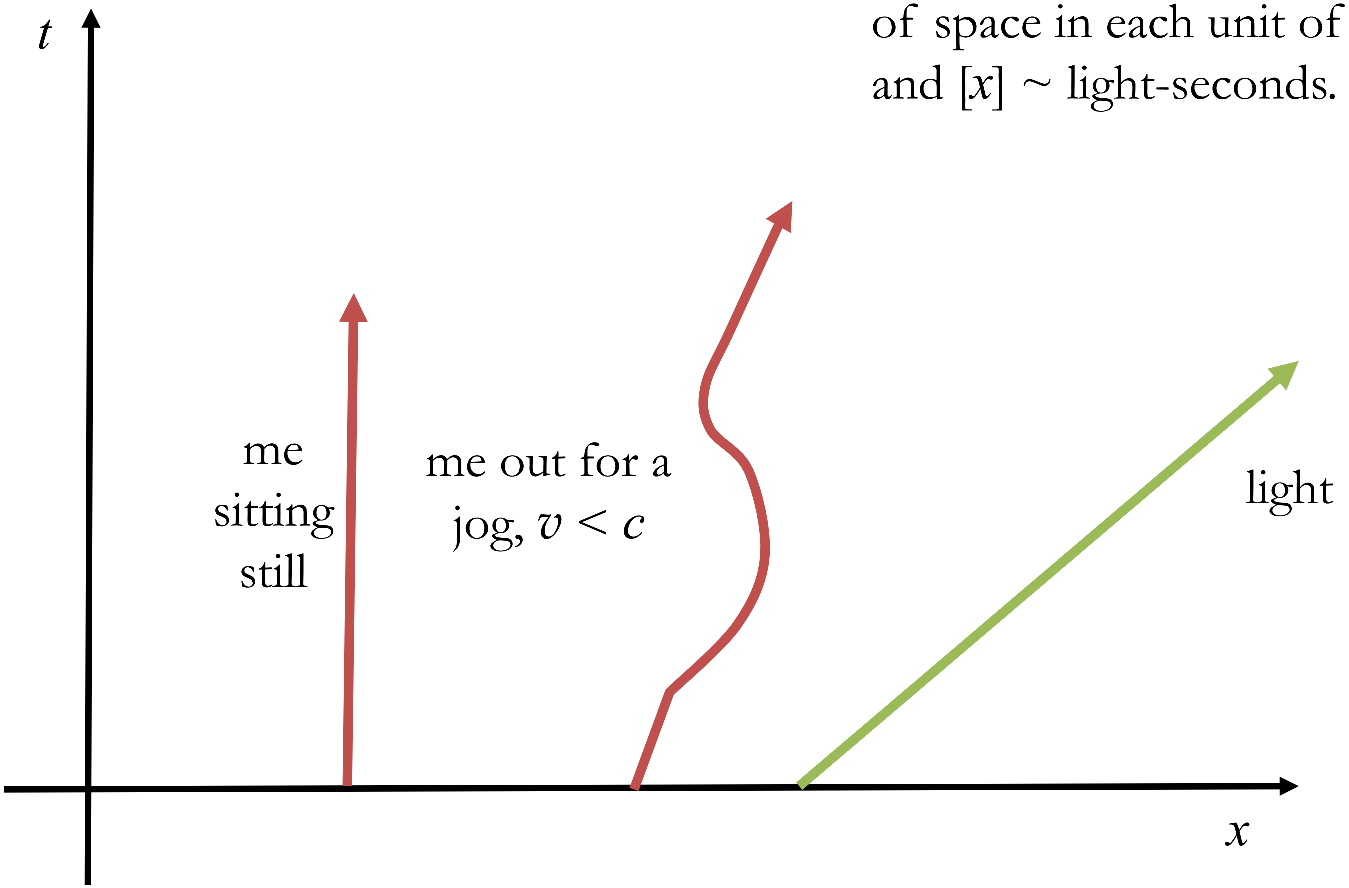
Once Minkowski did read Einstein's paper, he became convinced that Einstein had done a poor job, making things unnecessarily complicated, and missing the *real* point. So he reformulated the work in his own way in 1908.

Minkowski specialized in *geometry*. He had even written a book (first published in 1896) on *The Geometry of Numbers*, in which he had cast the abstract subject of number theory into geometrical terms.

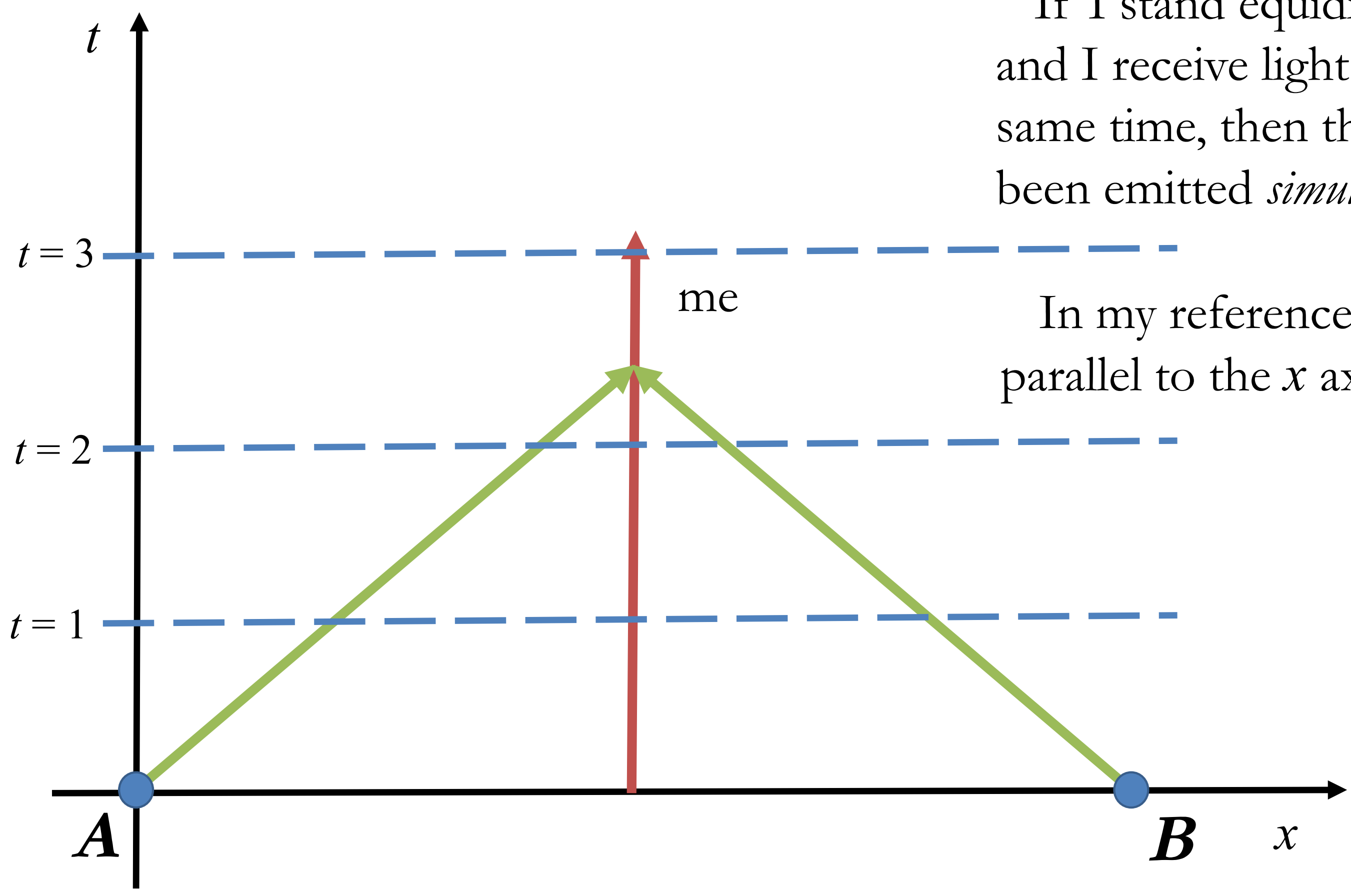
When he turned to Einstein's work on the electrodynamics of moving bodies, he did so as a *geometer*, rather than as a physicist who focused on performing measurements or a philosopher interested in Machian positivism.

Worldlines

Use coordinates such that light travels one unit of space in each unit of time, e.g. $[t]$ ~ seconds and $[x]$ ~ light-seconds. (In effect, we scale $c = 1$.)



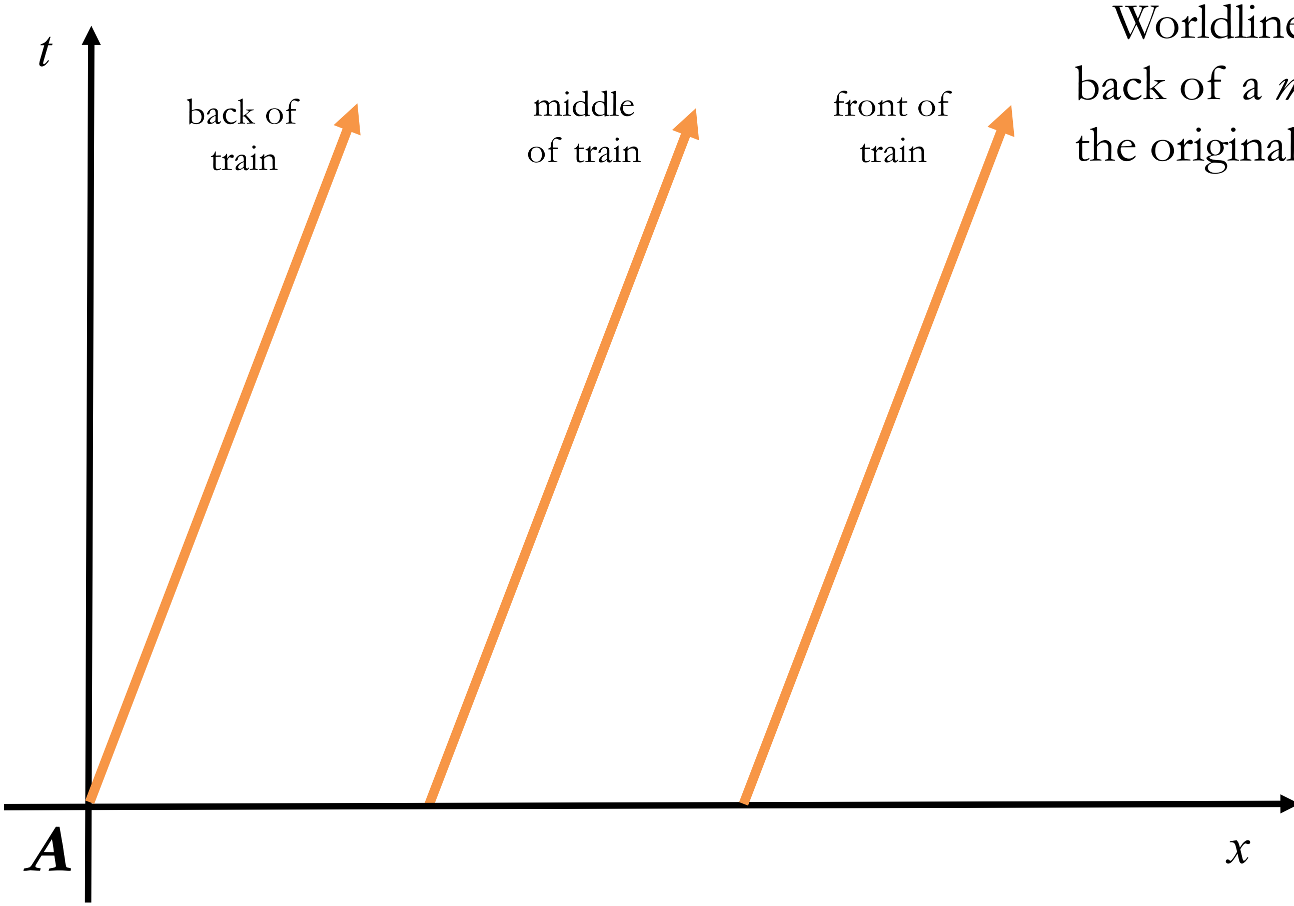
Lines of Simultaneity



If I stand equidistant from locations ***A*** and ***B*** and I receive light signals from ***A*** and ***B*** at the same time, then the light signals must have been emitted *simultaneously* (since c = constant).

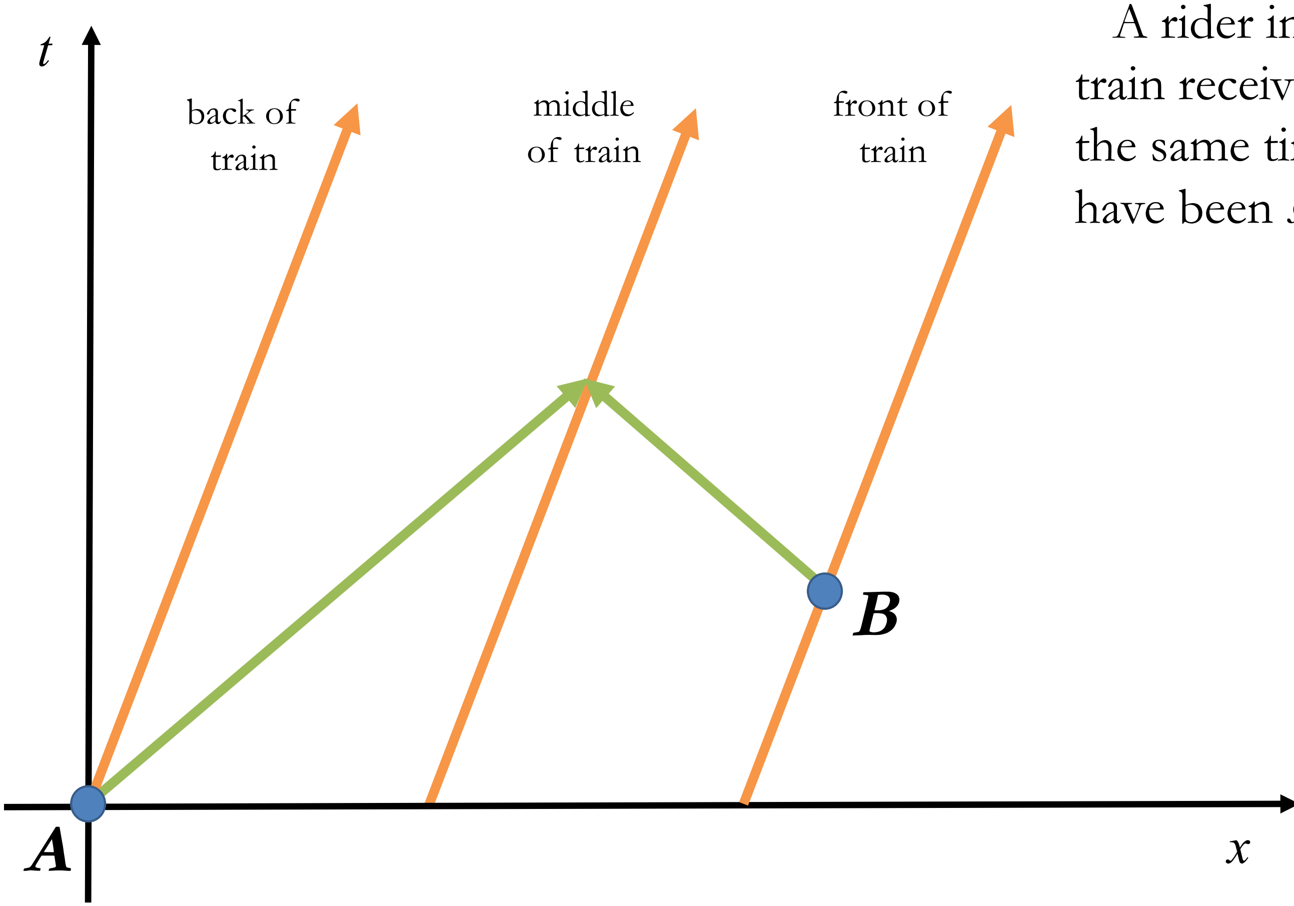
In my reference frame, *lines of simultaneity* are parallel to the x axis.

Lines of Simultaneity



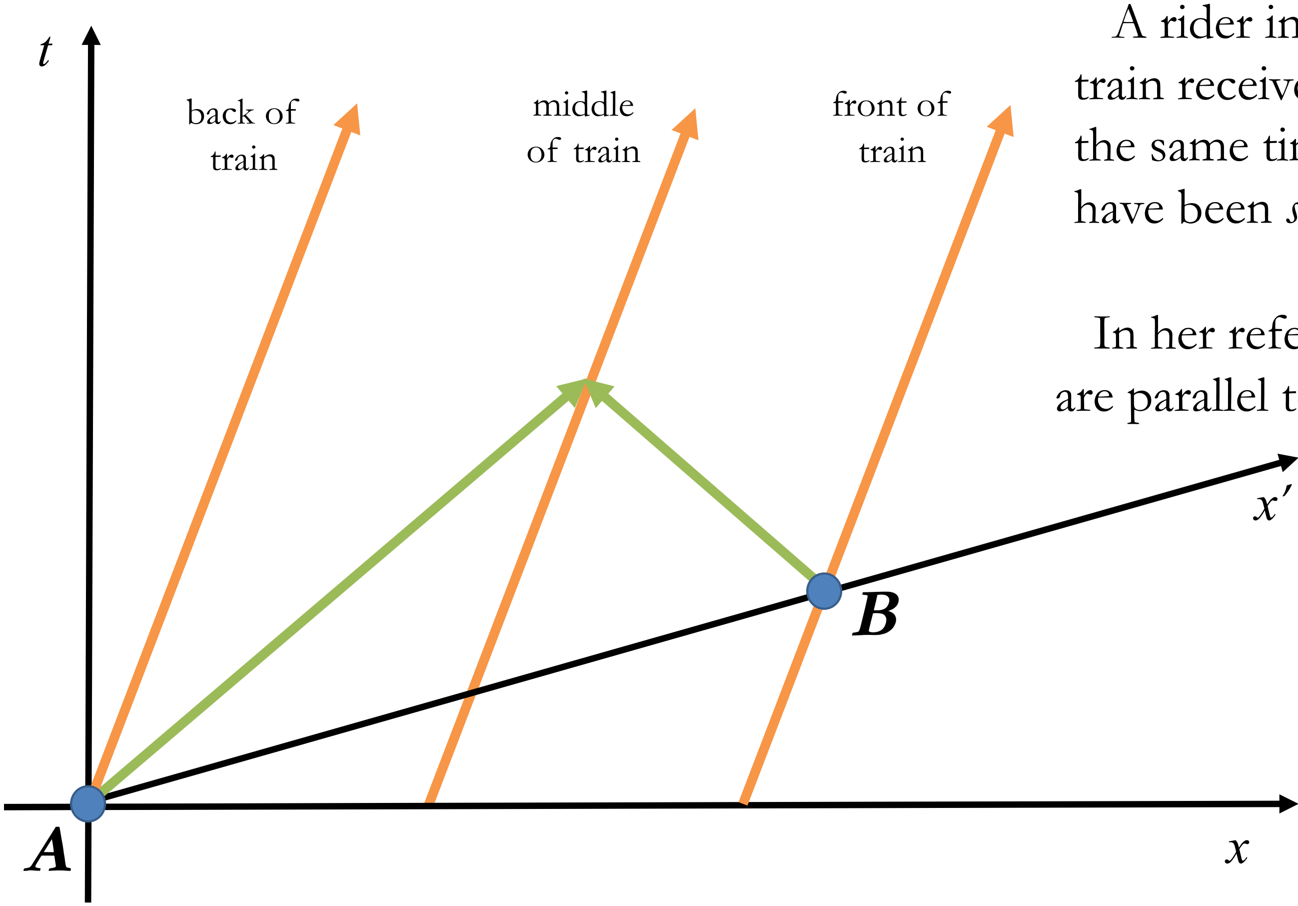
Worldlines for the front, middle, and back of a *moving* train, as observed within the original coordinate system.

Lines of Simultaneity



A rider in the middle of the (moving) train receives light signals from ***A*** and ***B*** at the same time. So the events ***A*** and ***B*** must have been *simultaneous* (since c = constant).

Lines of Simultaneity



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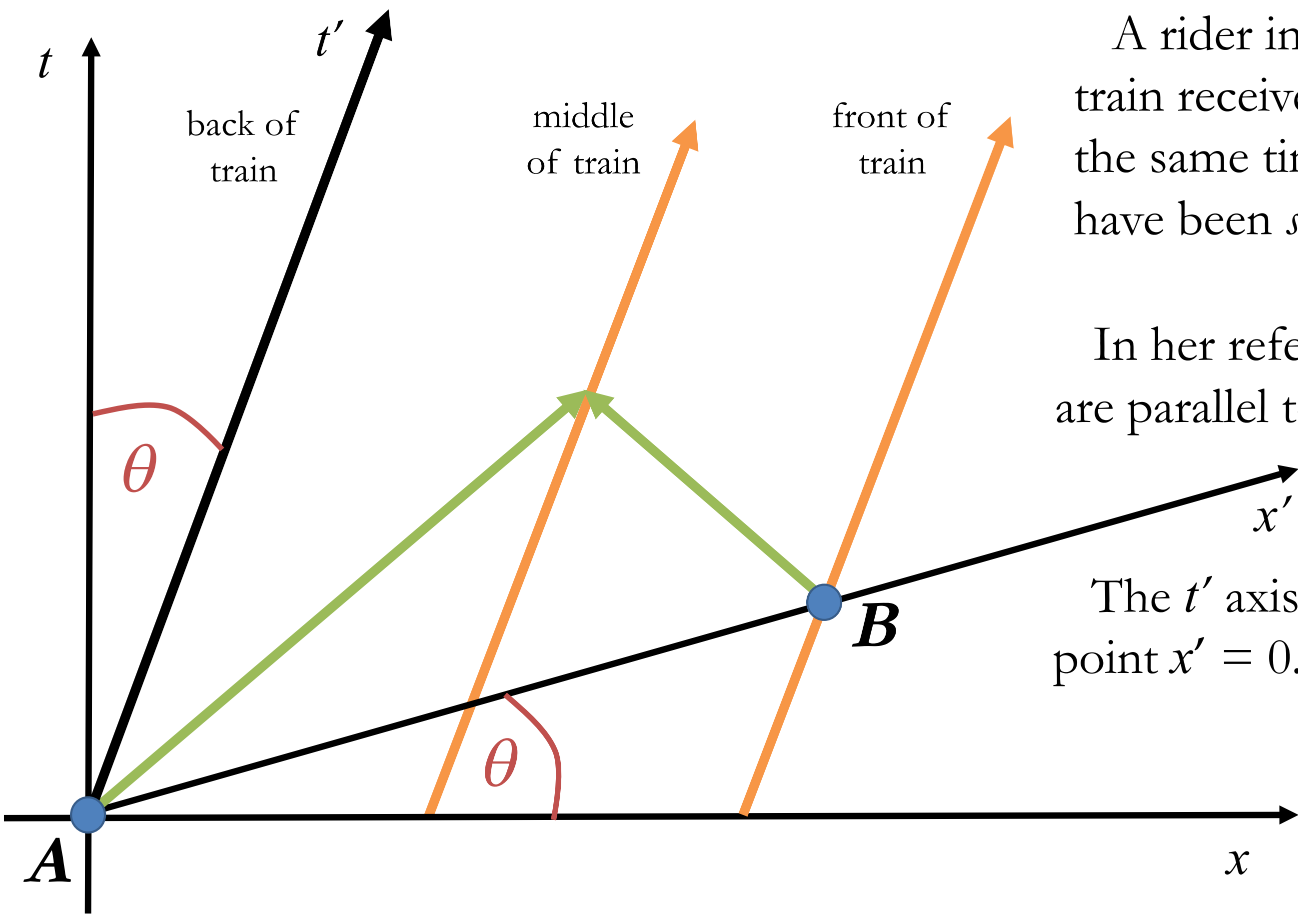

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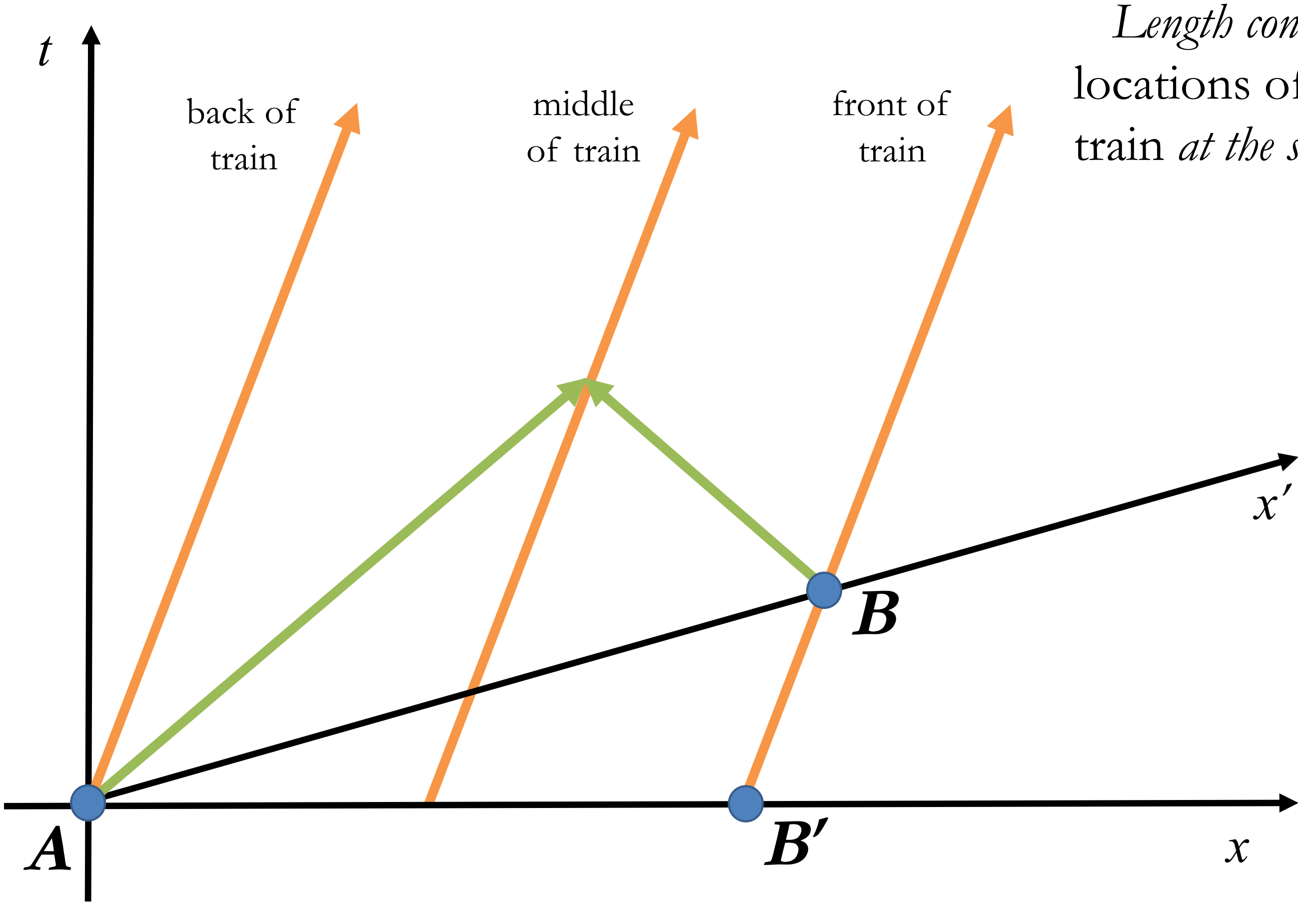
In her reference frame, *lines of simultaneity* are parallel to the x' axis.

The t' axis is just the *worldline* of the point $x' = 0$.

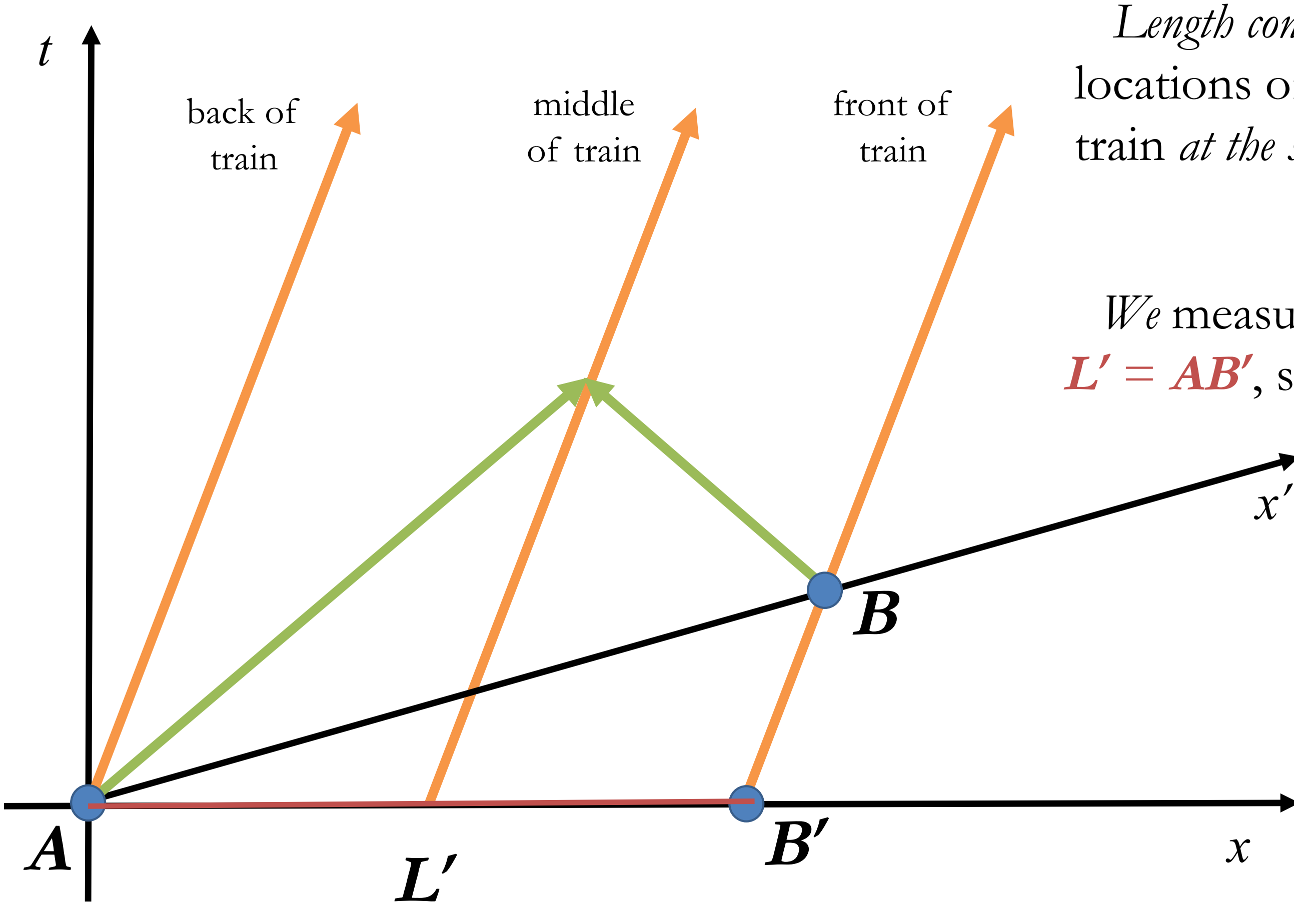
The angle θ between the x and x' axes and between the t and t' axes is the *same*: $\tan\theta = v/c$.

Lines of Simultaneity



Length contraction: we must measure the locations of the front and the back of the train *at the same time*.

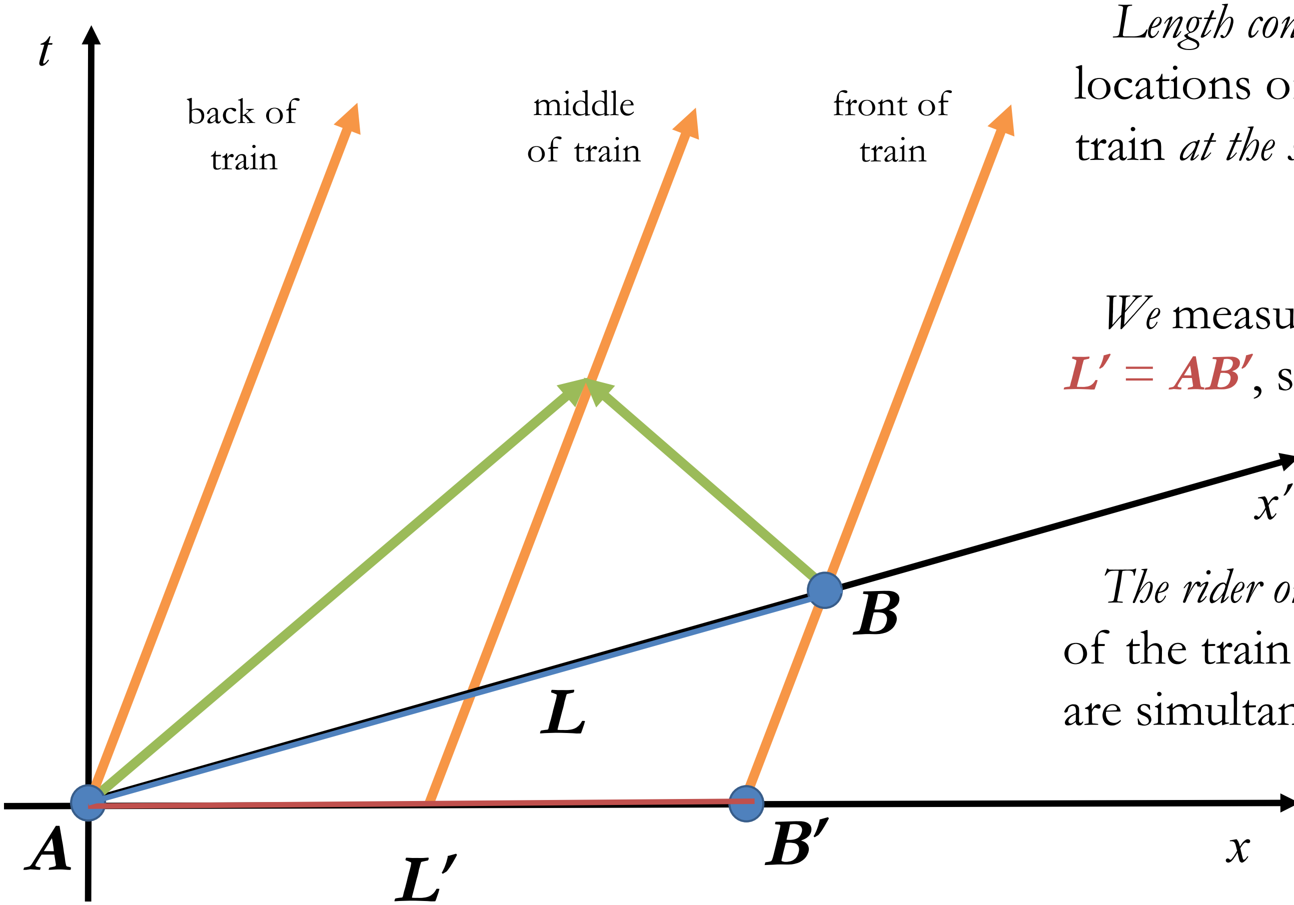
Lines of Simultaneity



Length contraction: we must measure the locations of the front and the back of the train *at the same time*.

We measure the length of the train to be ***L′* = *AB′***, since ***A*** and ***B′*** are simultaneous.

Lines of Simultaneity



Length contraction: we must measure the locations of the front and the back of the train *at the same time*.

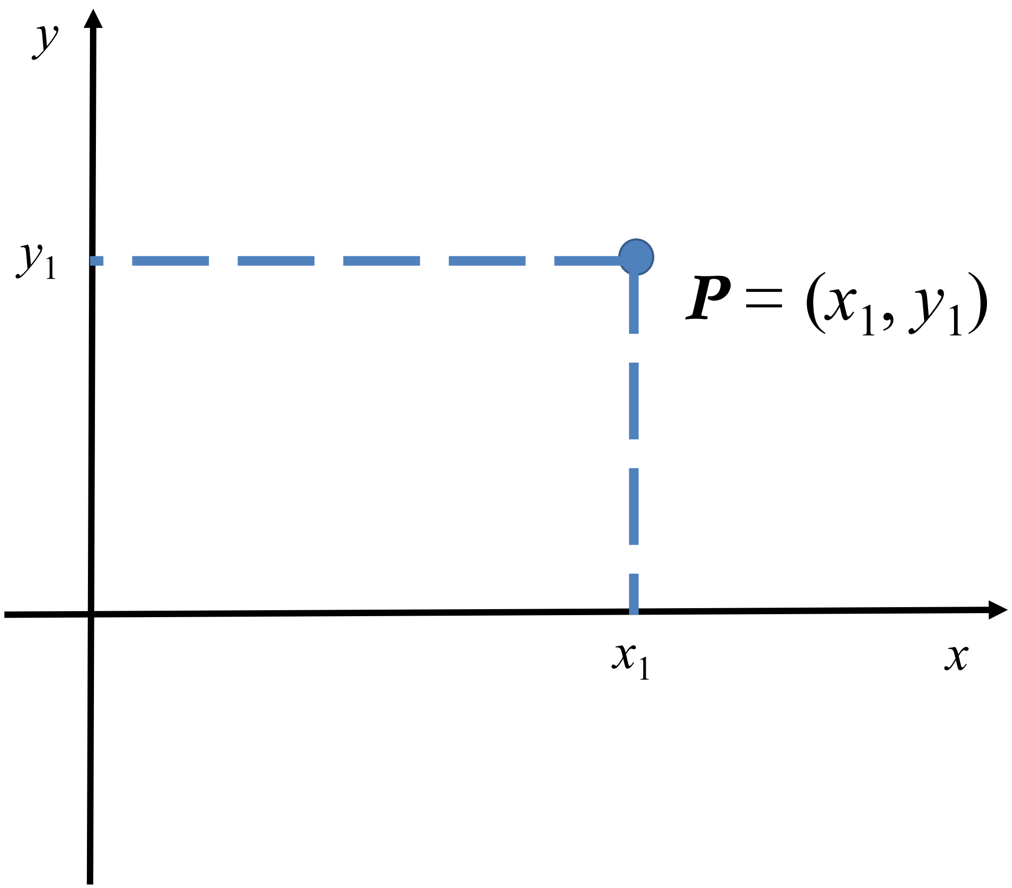
We measure the length of the train to be $\boldsymbol{L' = AB'}$, since $\boldsymbol{A}$ and $\boldsymbol{B'}$ are simultaneous.

The rider on the train measures the length of the train to be $\boldsymbol{L = AB}$, since $\boldsymbol{A}$ and $\boldsymbol{B}$ are simultaneous.

$$\boldsymbol{L' = L/\gamma}$$

To Minkowski, length contraction *was merely a geometrical effect* of projecting onto appropriate axes.

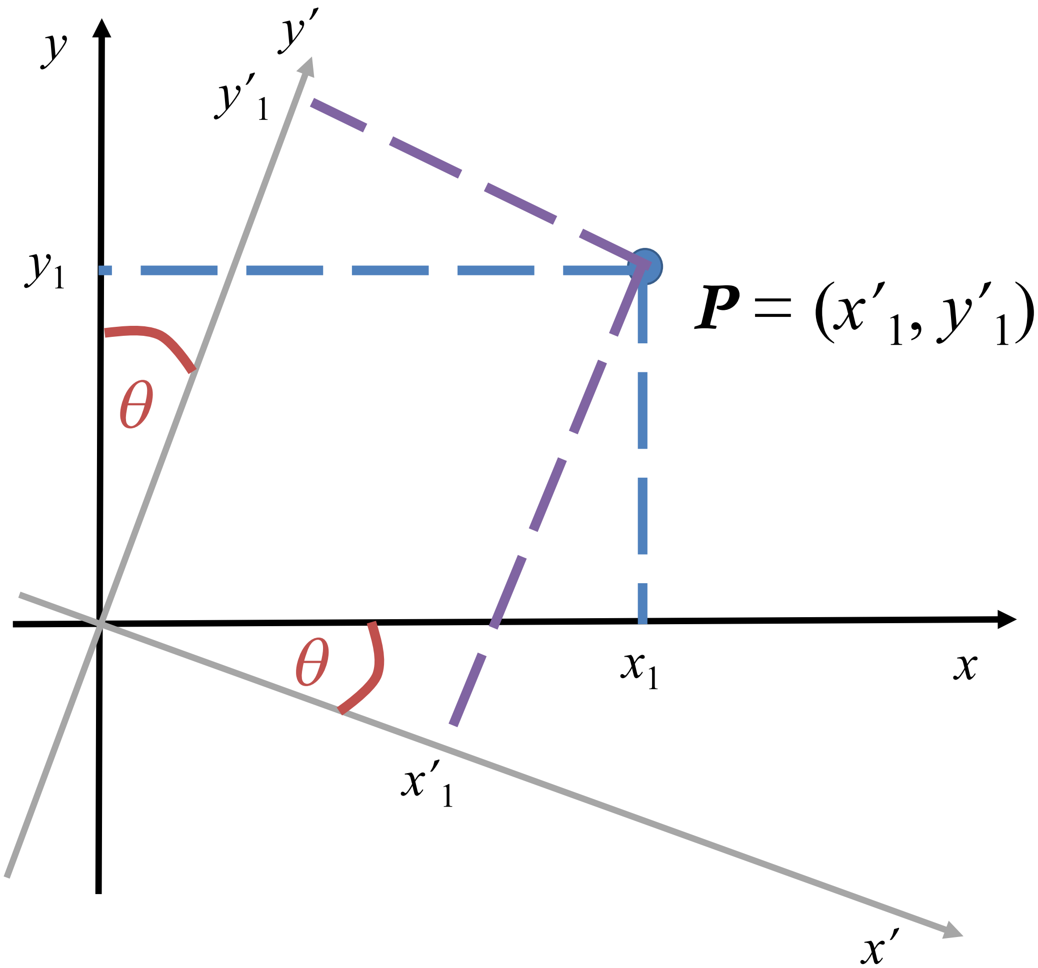
Rotations and Invariance



Using his new *space-time diagrams*, Minkowski next demonstrated that the *Lorentz transformation* was nothing but a geometric effect: a rotation in space-time.

First consider rotations in the x-y plane.

Rotations and Invariance



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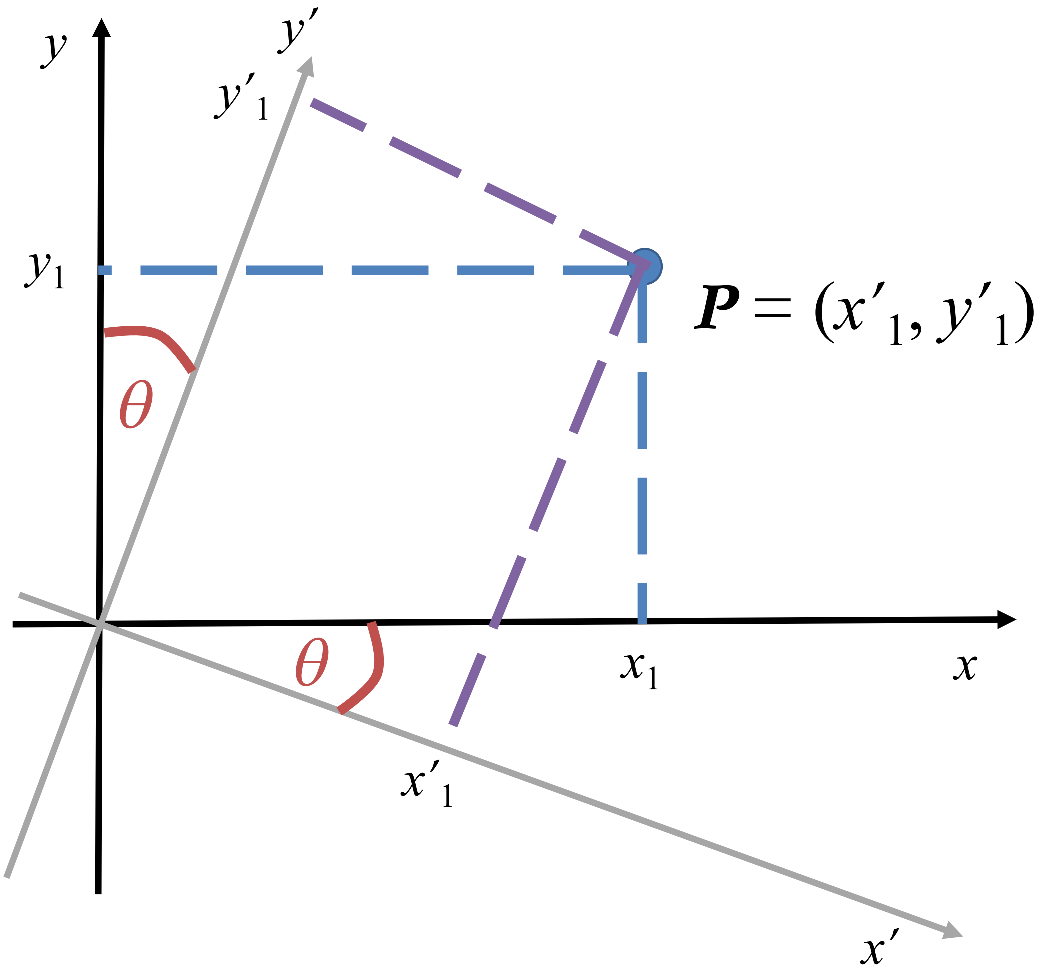

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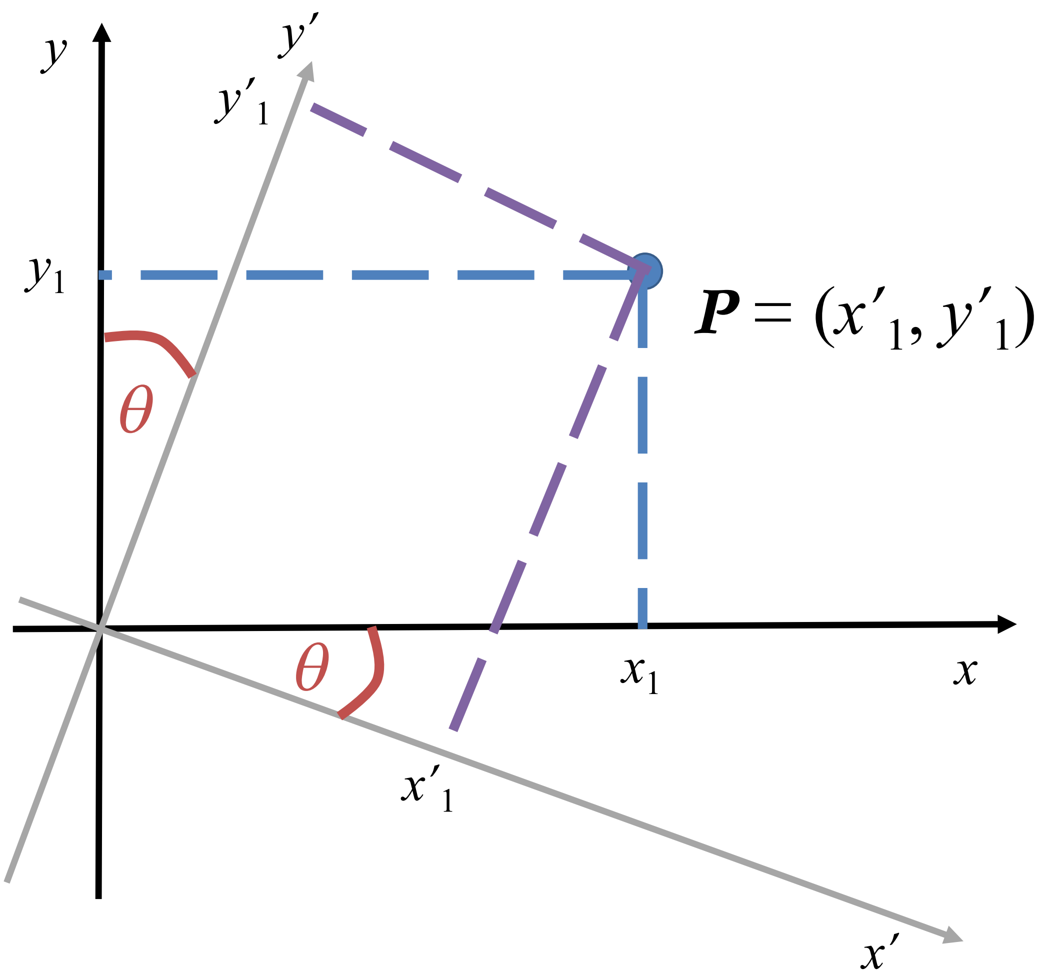
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = R(\theta) \begin{pmatrix} x \\ y \end{pmatrix}$$
$$= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Rotations and Invariance



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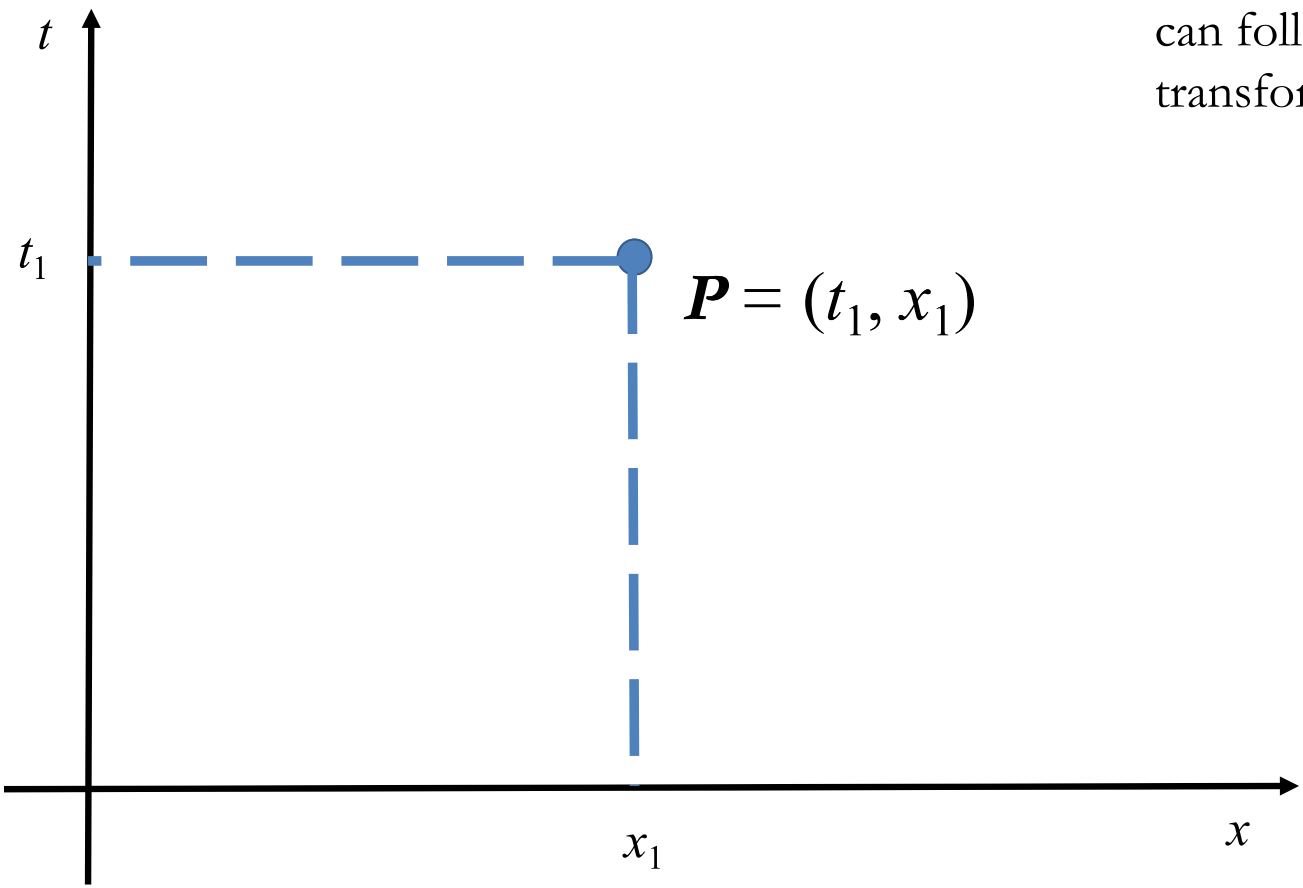
$$x' = x\cos\theta - y\sin\theta$$

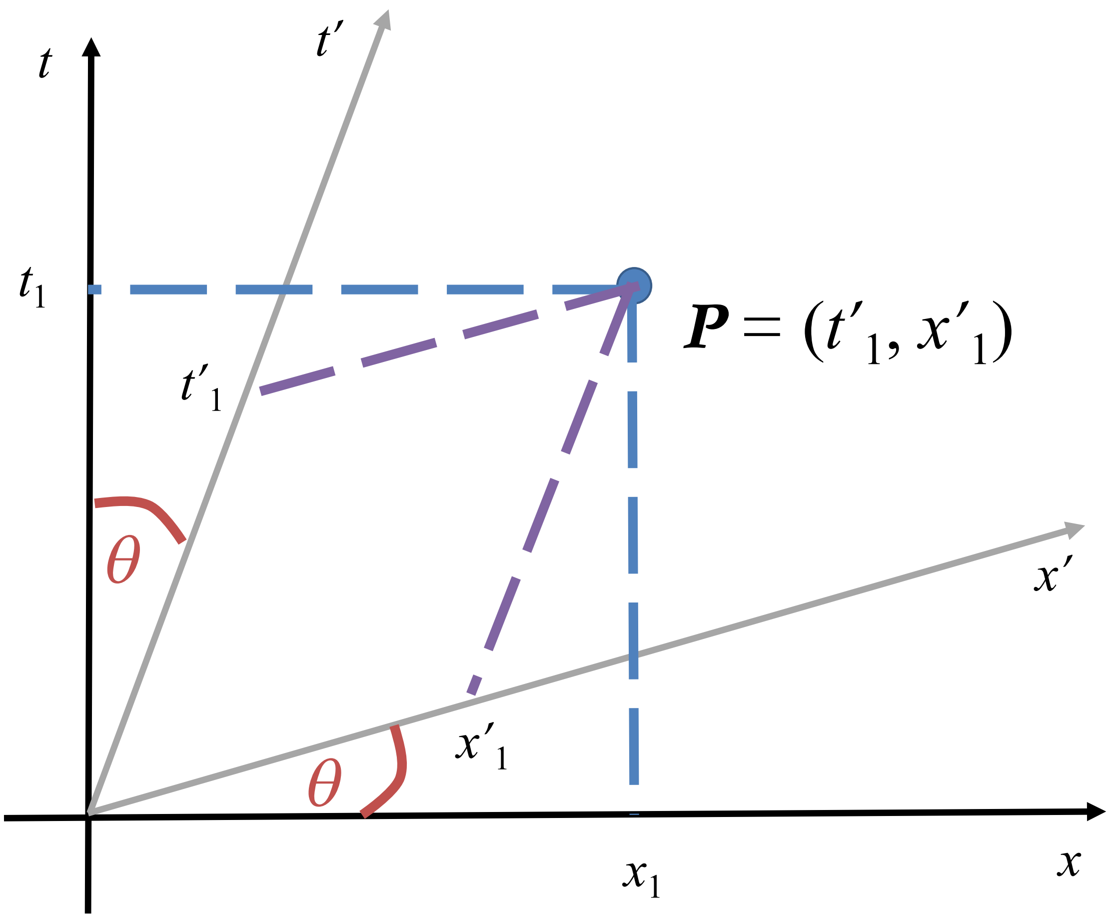
$$y' = x\sin\theta + y\cos\theta$$

Rotations and Invariance



Minkowski demonstrated that one can follow the same steps for transformations in *space-time*.

Rotations and Invariance



Recall: $\tan\theta = v/c$

Minkowski demonstrated that one can follow the same steps for transformations in *space-time*.

$$\begin{pmatrix} t' \\ x' \end{pmatrix} = \Lambda(v) \begin{pmatrix} t \\ x \end{pmatrix}$$

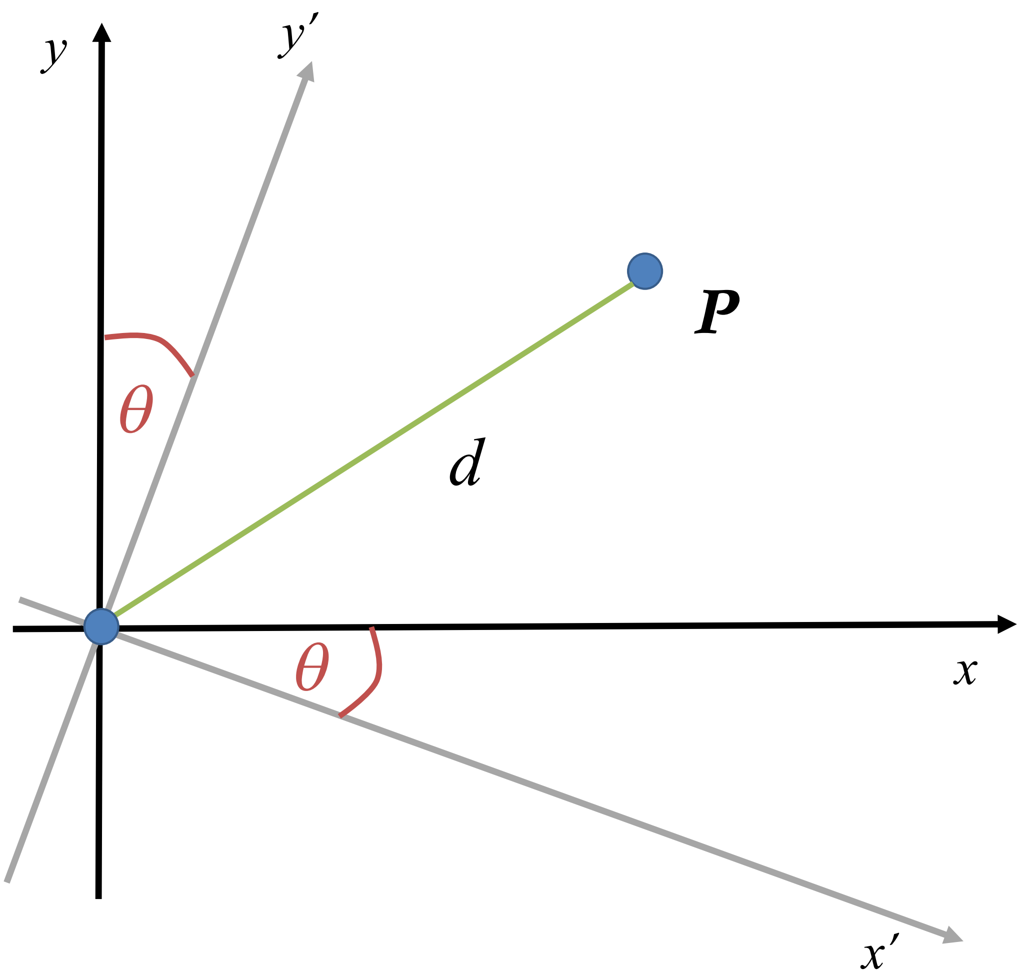
$$= \begin{pmatrix} \gamma & \gamma v/c^2 \\ \gamma v & \gamma \end{pmatrix} \begin{pmatrix} t \\ x \end{pmatrix}$$

$$t' = \gamma \left(t + \frac{vx}{c^2} \right)$$

$$x' = \gamma \left(x + vt \right)$$

To Minkowski, the *Lorentz transformation* was nothing but a geometrical rotation in space-time.

Rotations and Invariance



To Minkowski, the geometrical approach revealed an even more important lesson: *even amid rotations, some quantity should remain invariant.*

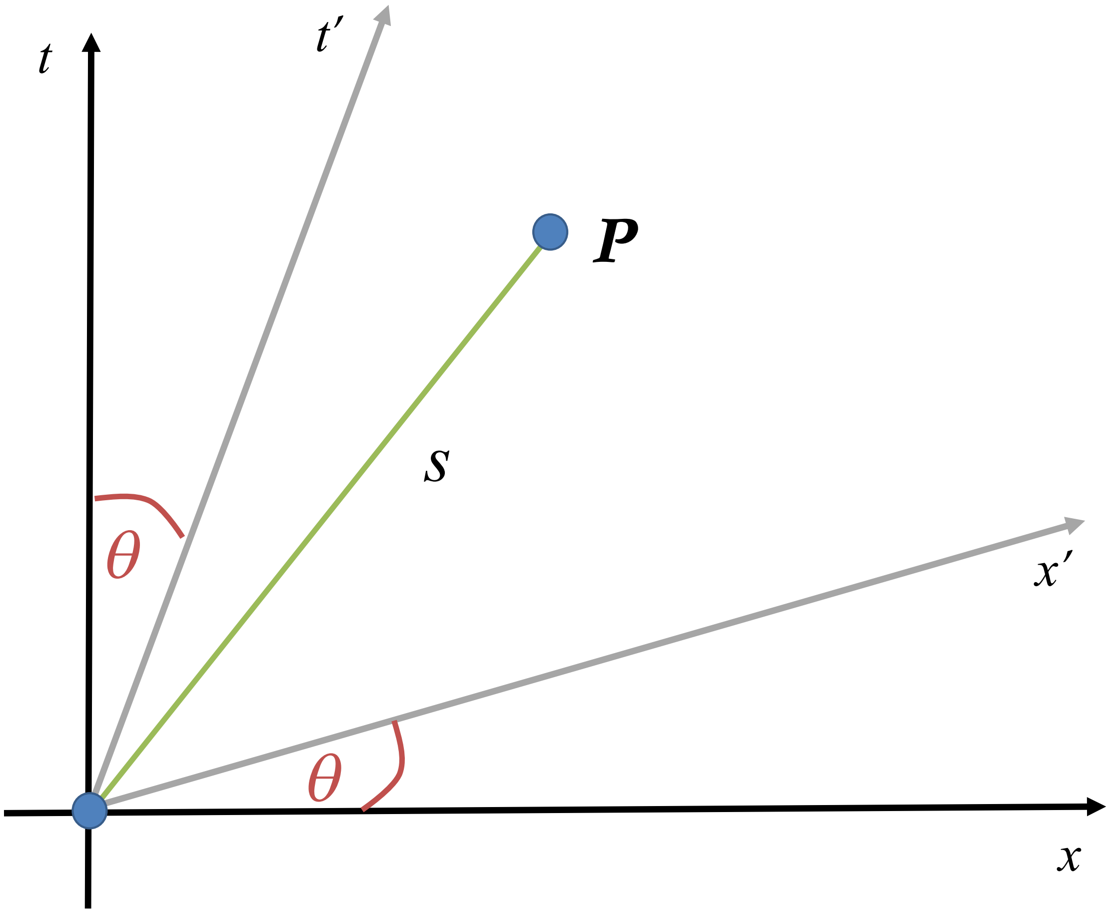
In the Euclidean plane, the *distance* between points does not change, even if one rotates from (x,y) to (x',y'):

$$x' = x\cos\theta - y\sin\theta$$

$$y' = x\sin\theta + y\cos\theta$$

$$\begin{aligned} d^2 &= (\Delta x)^2 + (\Delta y)^2 \\ &= (\Delta x')^2 + (\Delta y')^2 \end{aligned}$$

Rotations and Invariance

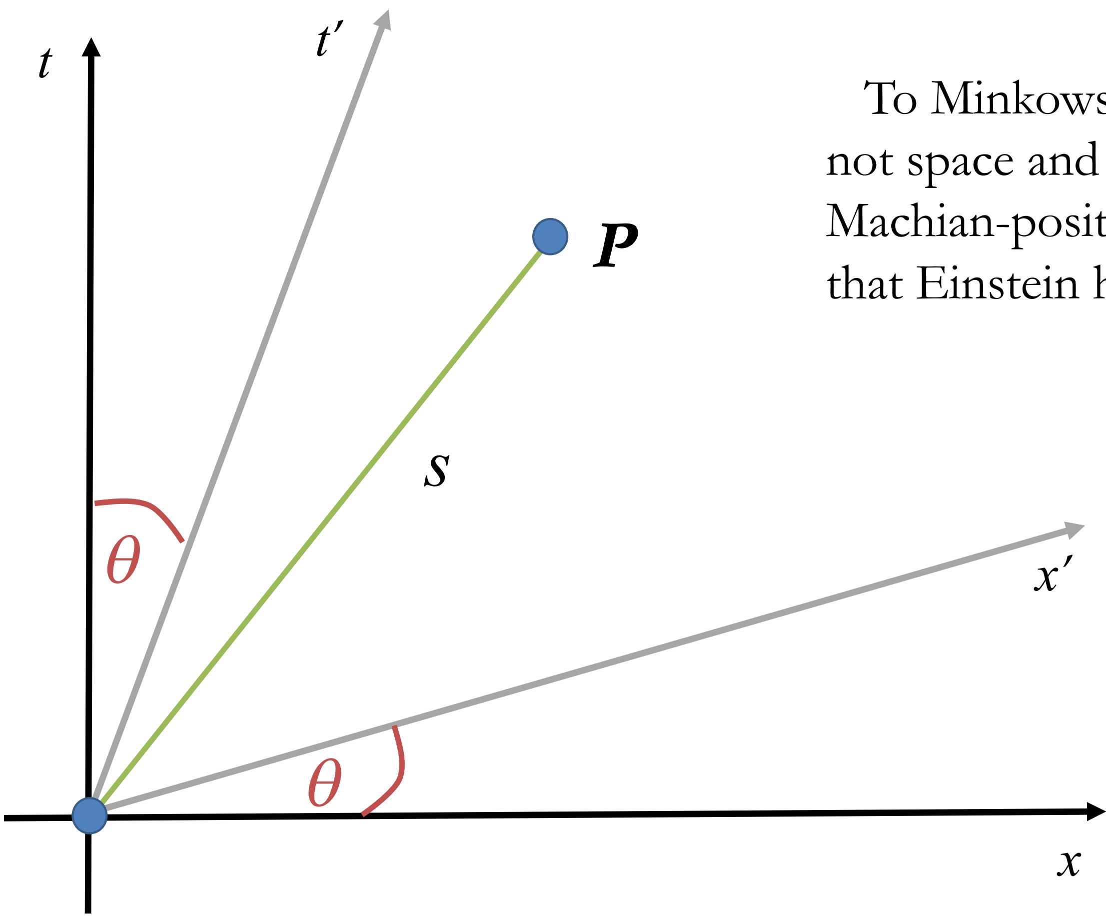


Minkowski demonstrated that a *space-time interval*, s, remained *invariant* under the Lorentz transformations:

$$t' = \gamma\left(t + \frac{vx}{c^2}\right)$$
$$x' = \gamma\left(x + vt\right)$$

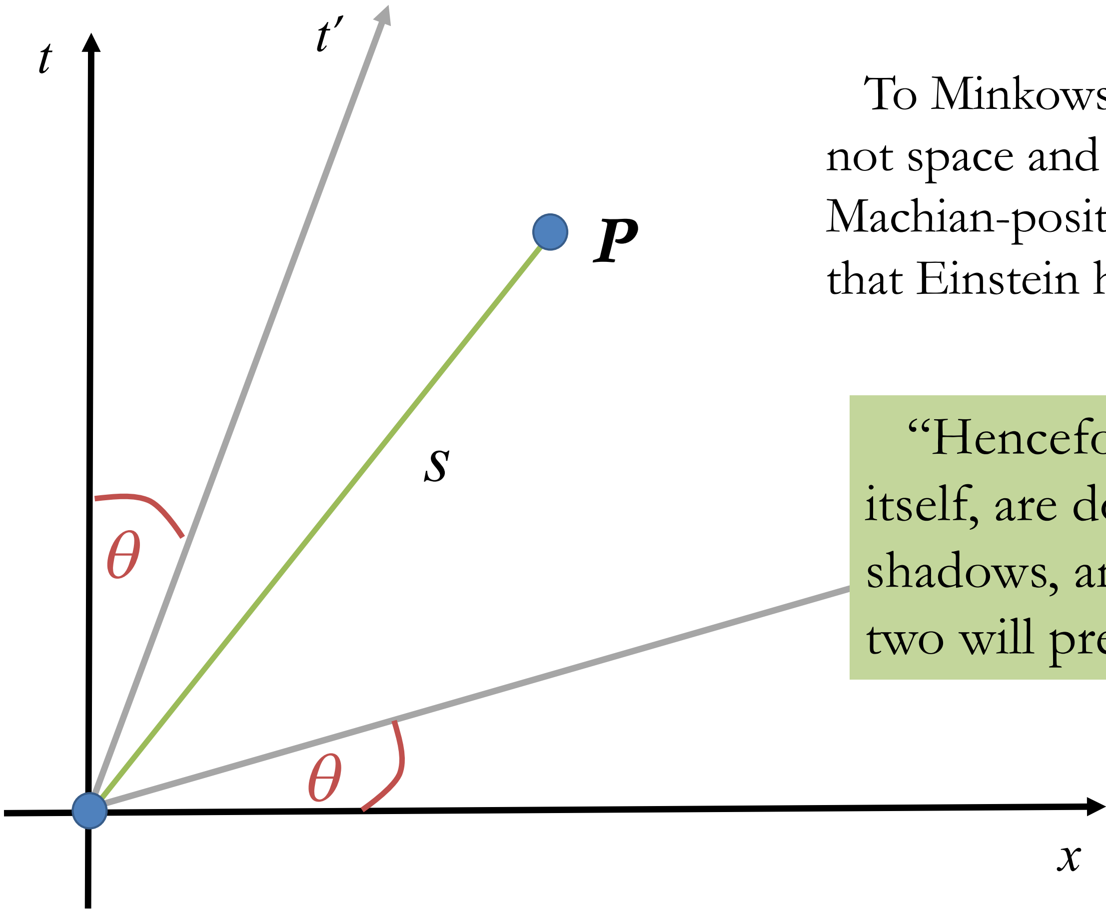
$$s^2 = c^2\left(\Delta t\right)^2 - \left(\Delta x\right)^2$$
$$= c^2\left(\Delta t'\right)^2 - \left(\Delta x'\right)^2$$

Rotations and Invariance



To Minkowski, *space-time* was the ultimate reality — not space and time separately, and certainly not the Machian-positivist descriptions of measurements that Einstein had belabored in his paper.

Rotations and Invariance

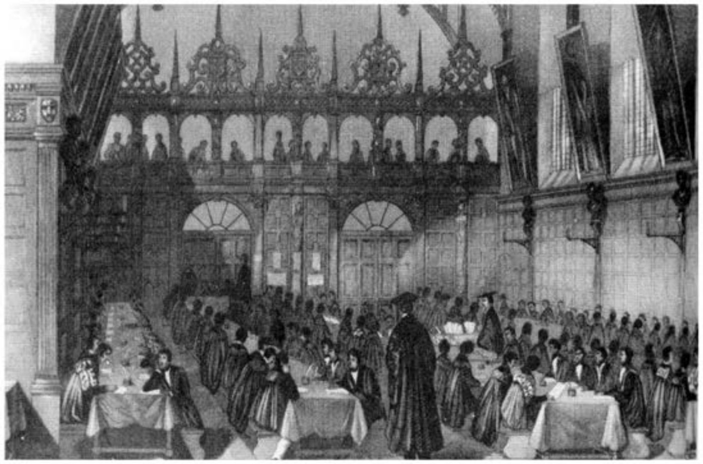


To Minkowski, *space-time* was the ultimate reality — not space and time separately, and certainly not the Machian-positivist descriptions of measurements that Einstein had belabored in his paper.

"Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve independence." (1908)

Questions?

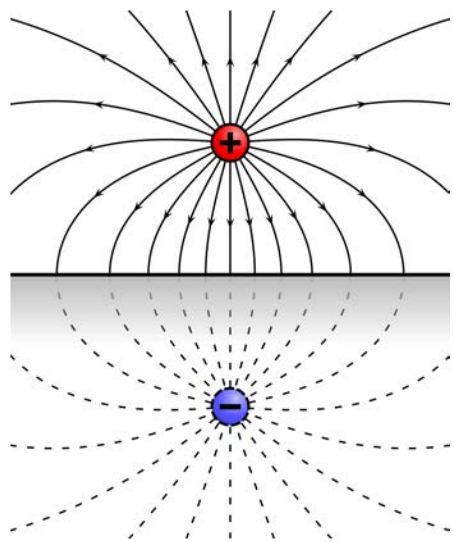
Cambridge Wranglers and Relativity



Written "Tripos" examination at Cambridge, mid-19th century

Much like Minkowski, other researchers who encountered Einstein's 1905 paper read it from within their own "reference frames." For Cambridge Wranglers, Einstein's work provided some exciting techniques for (further) exploring the behavior of classes of differential equations and their solutions.

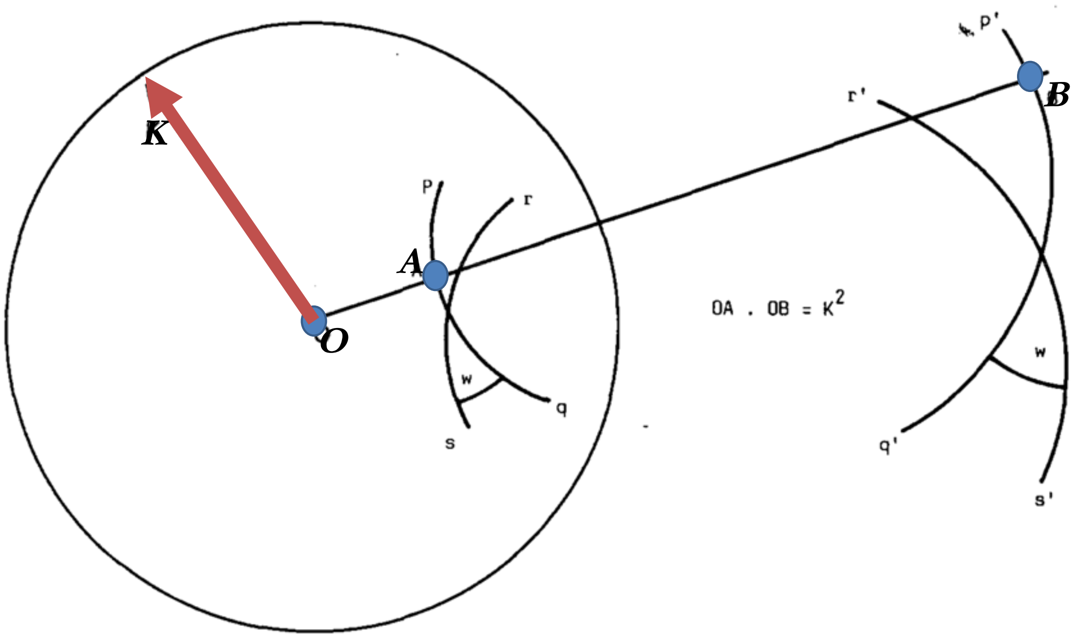
"*Image charges*" in electrostatics: solve for the field lines of a charge near a grounded conducting plate.



field lines must intersect the plate at right angles

One can find the exact solution by *replacing* the conducting plate with an "image charge" placed an equal distance *below* the plate.

Cambridge Wranglers and Relativity

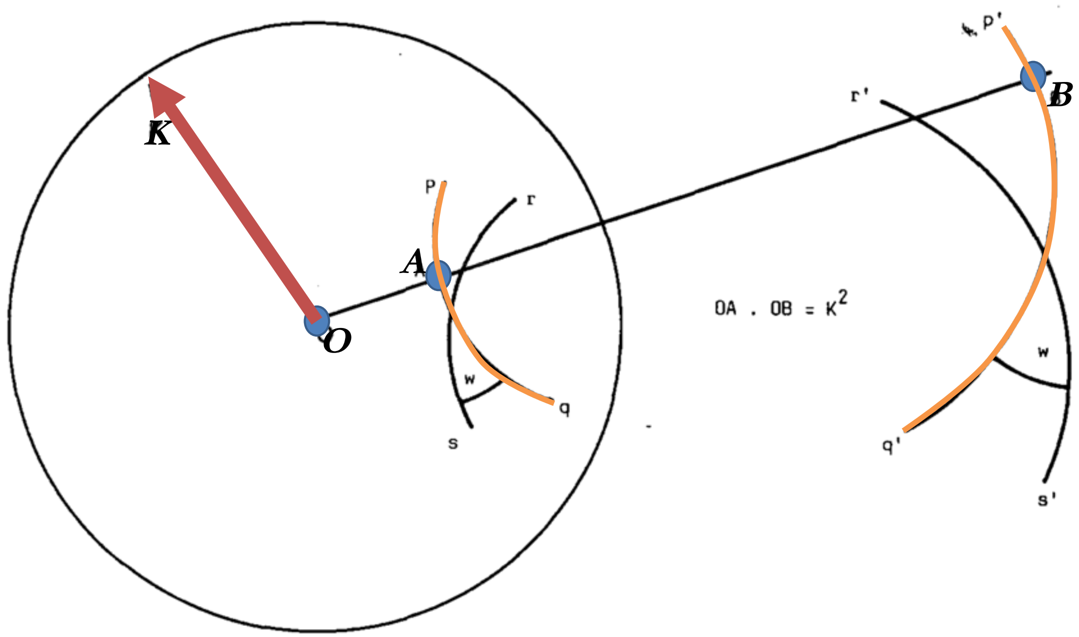


Method of "inversion": map a difficult problem into a simpler one.

Any point (such as ***A***) inside the circle of radius ***K*** may be mapped to an "inverse point" ***B*** by requiring

$$\boldsymbol{OA} \times \boldsymbol{OB} = \boldsymbol{K}^2$$

Cambridge Wranglers and Relativity



Method of "inversion": map a difficult problem into a simpler one.

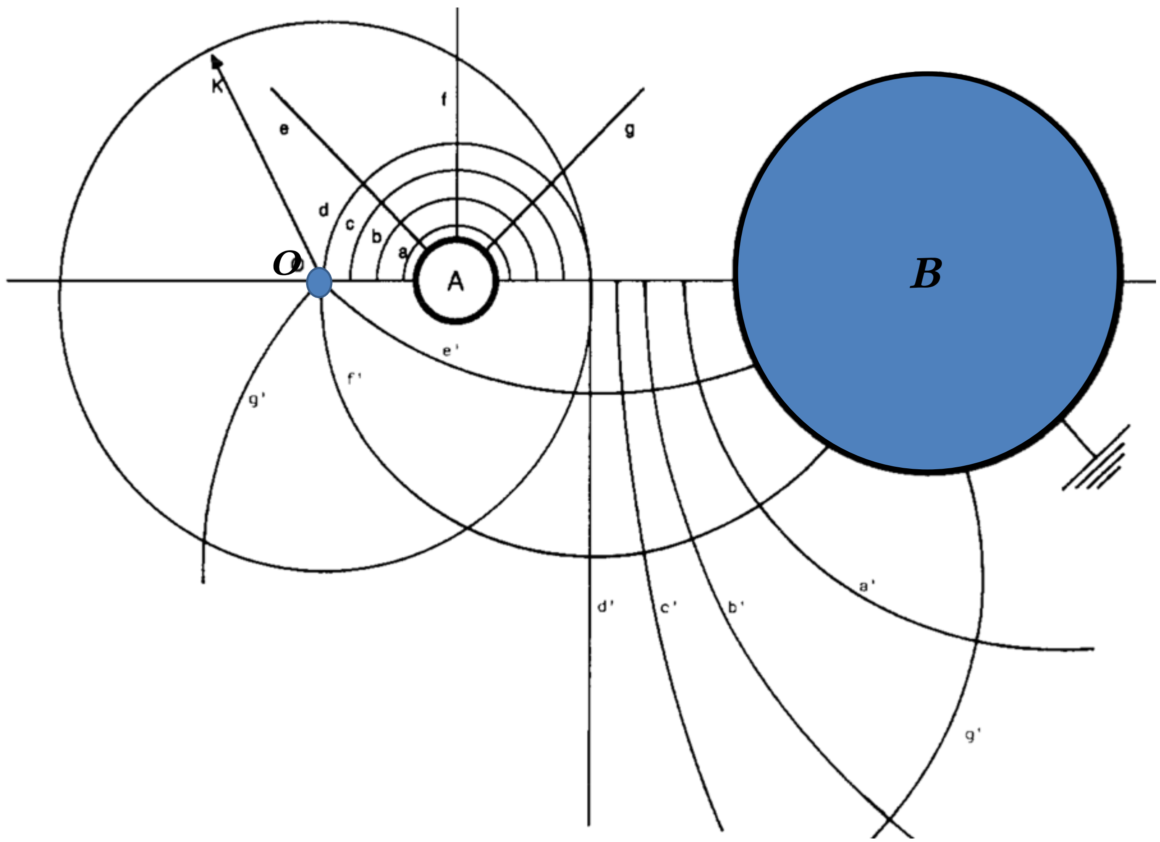
Any point (such as ***A***) inside the circle of radius ***K*** may be mapped to an "inverse point" ***B*** by requiring

$$OA \times OB = K^2$$

If the point ***A*** moves along the curve ***pq***, then the inverse point ***B*** moves along the curve ***p′q′***.

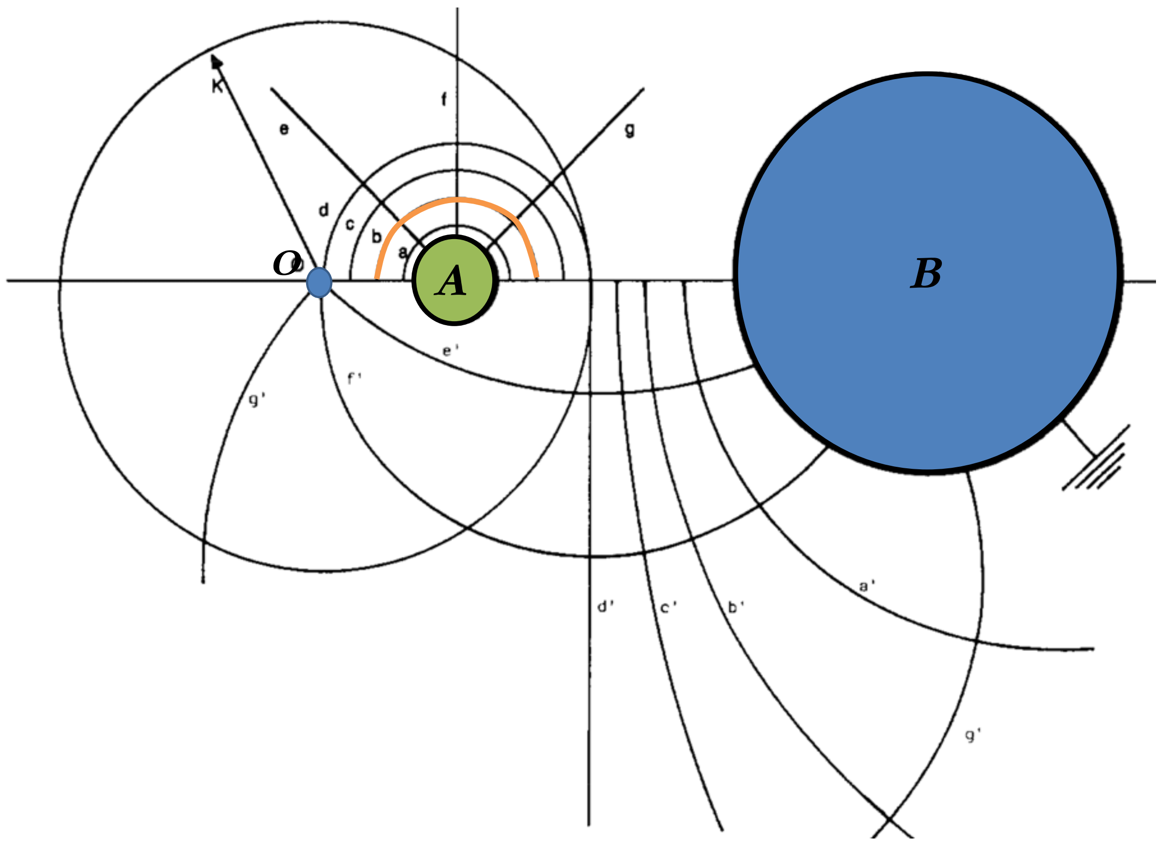
Such a transformation *preserves angles*, but not lengths: "conformal transformation."

Cambridge Wranglers and Relativity



If we want to solve for the field lines and equipotential surfaces due to a charge at location ***O*** near a grounded conducting sphere centered at ***B***, we may use inversion to treat a simpler problem:

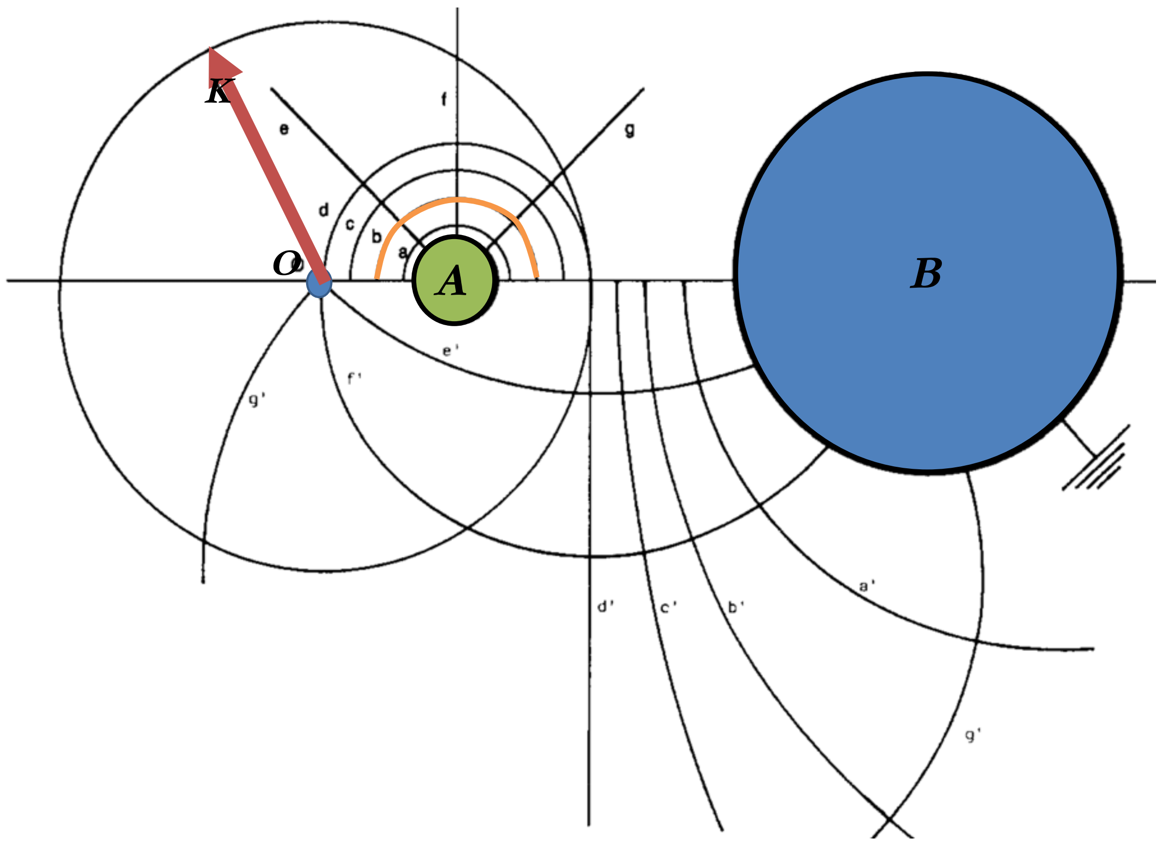
Cambridge Wranglers and Relativity



If we want to solve for the field lines and equipotential surfaces due to a charge at location ***O*** near a grounded conducting sphere centered at ***B***, we may use inversion to treat a simpler problem:

Consider a charged conducting sphere centered at ***A***. Draw the field lines and equipotential surfaces for the simple problem.

Cambridge Wranglers and Relativity



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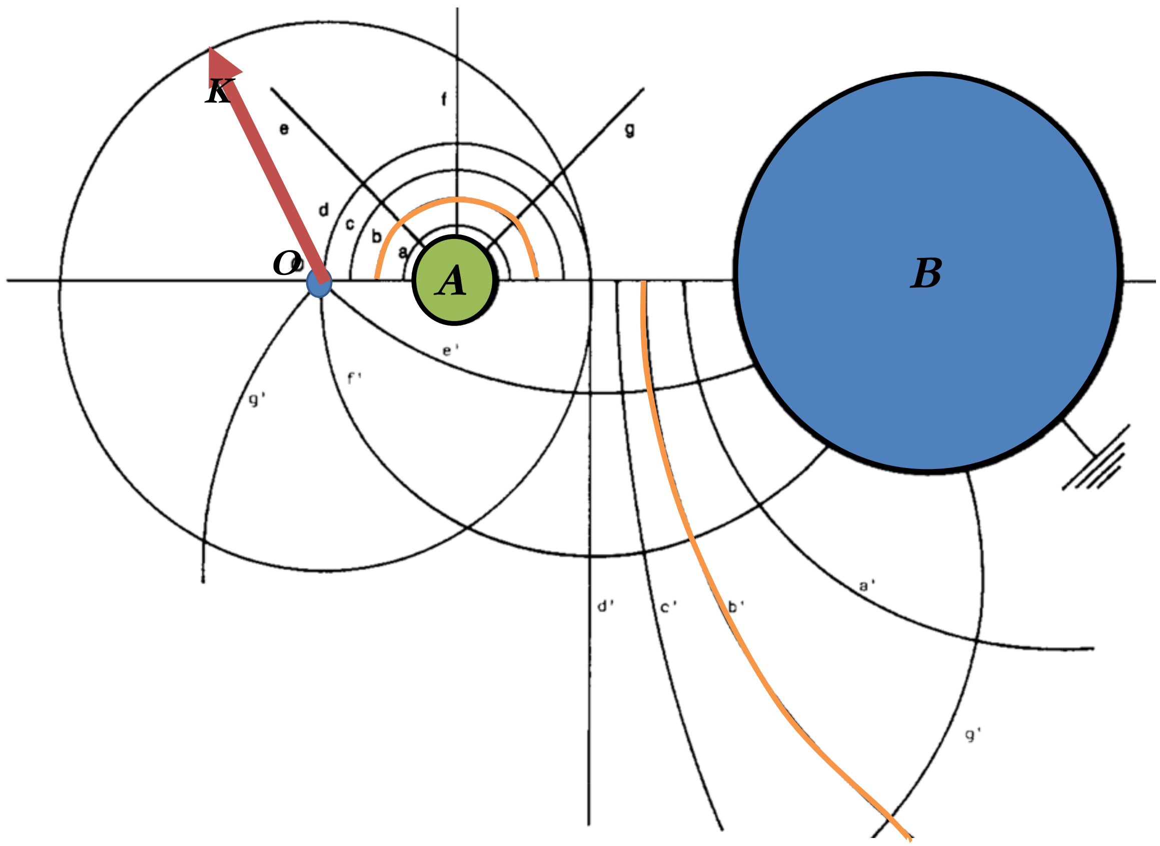
Consider a charged conducting sphere centered at ***A***. Draw the field lines and equipotential surfaces for the simple problem. Enclose ***A*** in a sphere of radius ***K***.

Cambridge Wranglers and Relativity



If we want to solve for the field lines and equipotential surfaces due to a charge at location ***O*** near a grounded conducting sphere centered at ***B***, we may use inversion to treat a simpler problem:

Consider a charged conducting sphere centered at ***A***. Draw the field lines and equipotential surfaces for the simple problem. Enclose ***A*** in a sphere of radius ***K***. Map the curves to the original system involving ***B***.

(And do all this on a timed exam that determined your graduation rank!)

Cambridge Wranglers and Relativity

The method of inversion and conformal transformations had become *routine* for Cambridge Wranglers by 1900. These techniques applied to *electrostatics*, that is, situations in which the electric potential did not vary over time:

$$\nabla^2 \phi(x, y, z) = 0$$

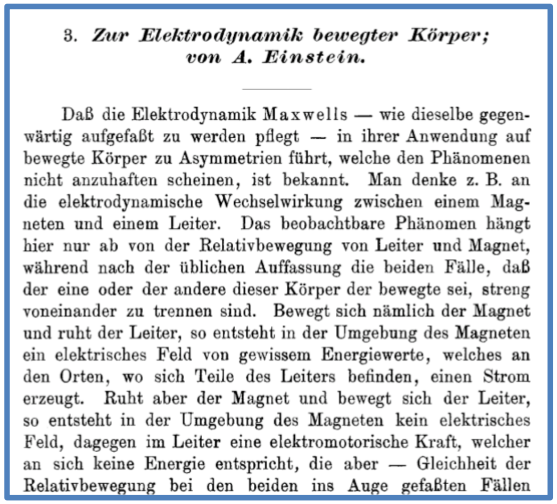
Ebeneezer Cunningham and *Harry Bateman* aimed to generalize those techniques to *electrodynamics*. To them, Einstein's paper offered ideas about how to identify all transformations $\Lambda(v)$ that would leave the equation

$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right] \phi(t, x, y, z) = 0$$

invariant. Then they could use the *inversion* technique to produce *new solutions* $\phi'(t', x', y', z')$. They sought to prove they had found the *most general class* of transformations $\Lambda(v)$.

For the Wranglers, *that's* what relativity was all about — not the ether, not Machian positivism, not Minkowski's space-time ...

Reception of Relativity: Summary



3. *Zur Elektrodynamik bewegter Körper;*
von A. Einstein.

Daß die Elektrodynamik Maxwells — wie dieselbe gegenwärtig aufgefaßt zu werden pflegt — in ihrer Anwendung auf bewegte Körper zu Asymmetrien führt, welche den Phänomenen nicht anzuhaften scheinen, ist bekannt. Man denke z. B. an die elektrodynamische Wechselwirkung zwischen einem Magneten und einem Leiter. Das beobachtbare Phänomen hängt hier nur ab von der Relativbewegung von Leiter und Magnet, während nach der üblichen Auffassung die beiden Fälle, daß der eine oder der andere dieser Körper der bewegte sei, streng voneinander zu trennen sind. Bewegt sich nämlich der Magnet und ruht der Leiter, so entsteht in der Umgebung des Magneten ein elektrisches Feld von gewissem Energiewerte, welches an den Orten, wo sich Teile des Leiters befinden, einen Strom erzeugt. Ruht aber der Magnet und bewegt sich der Leiter, so entsteht in der Umgebung des Magneten kein elektrisches Feld, dagegen im Leiter eine elektromotorische Kraft, welcher an sich keine Energie entspricht, die aber — Gleichheit der Relativbewegung bei den beiden ins Auge gefaßten Fällen

Researchers did not just read Einstein's paper and become devoted Einsteinians. *Most* physicists ignored the paper completely for 5-10 years; others assumed it was merely an elaboration of Lorentz's prior work.

Those few who *did* pay attention did so from within their own contexts. Different aspects of Einstein's paper seemed more or less relevant to them.

Minkowski re-interpreted Einstein's paper in terms of *space-time geometry*. Cunningham and Bateman re-interpreted the paper in terms of *properties of differential equations* and inversion properties of their solutions.

None of these readers seemed to care much about was most important to Einstein: that the ether was "merely superfluous," and that, following Ernst Mach, one should start with *kinematics* instead of *dynamics*. As we've seen before: *same equations, different meanings!*