22.101 Applied Nuclear Physics (Fall 2006)
Lecture 1 (9/6/06)

Basic Nuclear Concepts

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References –

Table of Isotopes, C. M. Lederer and V. S. Shirley, ed. (Wiley & Sons, New York, 1978), 7th ed.
Table of Nuclides <http://atom.kaeri.re.kr/>
Physics Vade Mecum, H. L. Anderson, ed. (American Institute of Physics, New York, 1081).
P. Marmier and E. Sheldon, *Physics of Nuclei and Particles* (Academic Press, New York, 1969), vol. 1.

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General Remarks:

This subject treats foundational knowledge for all students in the Department of Nuclear Science and Engineering. Over the years 22.101 has evolved in the hands of several instructors, each of whom dealt with the subject contents in somewhat different fashions. Two topics, in particular, were not given the same treatment by the different faculty who have taught 22.101, quantum mechanics and the interaction of radiation with matter. There was also some difference in how much nuclear structure and nuclear models were taught from the perspective of a course on nuclear physics in a physics department.

In the present version of 22.101 we intend to emphasize the nuclear concepts, as opposed to traditional nuclear physics, essential for understanding nuclear radiations and their interactions with matter. The justification for is that we see our students as nuclear engineers rather than nuclear physicists. Nuclear engineers work with all kinds of nuclear devices, from fission and fusion reactors to accelerators and detection systems. In all these complex systems nuclear radiations play a central role. In generating nuclear radiations and using them for beneficial purposes, scientists and engineers must understand the properties of these radiations and how they interact with their surroundings. It is through the control of radiations interactions that we can develop new devices or optimize existing ones to make them more safe, powerful, durable, or economical. This is the simple reason why radiation interaction is the essence of 22.101.

Because nuclear physics is a very large subject and in view of our focus on radiation interactions, we will not be covering some of the standard material on nuclear structure and models that would be normally treated in a physics course. Students interested in these topics are encouraged to read up on your own. We should also note that in 22.101 we will study the different types of reactions as single-collision phenomena, described through variouscross sections, and leave the accumulated effects of many collisions to later subjects (22.105 and 22.106). Although we will not teach quantum mechanics by itself, it is used in several descriptions of nuclei, at a more introductory level than in 22.51 and 22.106.

Nomenclature:

$_z X^A$ denotes a nuclide, a specific nucleus with Z number of protons (Z = *atomic number*) and A number of nucleons (neutrons or protons). The symbol of nucleus is X which is either a single letter as in uranium U, or two letters as in copper Cu or plutonium Pu. There is a one-to-one correspondence between Z and X, so that specifying both is actually redundant (but helpful since not everyone remembers the atomic number of all the elements). The number of neutrons N of this nucleus is A – Z. Often it is sufficient to specify only X and A, as in U^{235}, if the nucleus is a familiar one (uranium is well known to have Z=92). The symbol A is called the *mass number* since knowing the number of nucleons one has an approximate idea of what is the mass of the particular nucleus. There exist several uranium nuclides with different mass numbers, such as U^{233}, U^{235}, and U^{238}. Nuclides with the same Z but different A are called *isotopes*. By the same token, nuclides with the same A but different Z are called *isobars*, and nuclides with same N but different Z are called *isotones*. *Isomers* are nuclides with the same Z and A in different excited states. For a compilation of the nuclides that are known, see the *Table of Nuclides* (Kaeri) for which a website is given in the References.

We are, in principle, interested in all the elements up to Z = 94 (plutonium). There are about 20 more elements which are known, most with very short lifetimes; these are of interest mostly to nuclear physicists and chemists, but not so much to nuclear engineers. While each element can have several isotopes of significant abundance, not

all the elements are of equal interest to us in this class. The number of nuclides we might encounter in our studies is probably no more than 20.

A great deal is known about the properties of nuclides. In addition to the *Table of Nuclides*, the student can consult the *Table of Isotopes*, cited in the References. It should be appreciated that the great interest in nuclear structure and reactions is not just for scientific knowledge alone; the fact that there are two applications that affects the welfare of our society – nuclear power and nuclear weapons – has everything to do with it.

We begin our studies with a review of the most basic physical attributes of nuclides to provide motivation and a basis to introduce what we want to accomplish in this course (see the Lecture Outline).

Basic Physical Attributes of Nuclides

Nuclear Mass

We adopt the unified scale where the mass of C^{12} is exactly 12. On this scale, one mass unit 1 mu ($C^{12} = 12$) = $M(C^{12})/12$ = 1.660420 x 10^{-24} gm (= 931.478 Mev), where $M(C^{12})$ is actual mass of the nuclide C^{12}. Studies of atomic masses by mass spectrograph shows that a nuclide has a mass nearly equal to the mass number A times the proton mass. Three important rest mass values, in mass and energy units, to keep handy are:

	mu [$M(C^{12}) = 12$]	Mev
electron	0.000548597	0.511006
proton	1.0072766	938.256
neutron	1.0086654	939.550

Reason we care about the mass is that it is *an indication of the stability* of the nuclide. One sees this directly from $E = Mc^2$, the higher the mass the higher the energy and the less stable is the nuclide (think of nuclide being in an excited state). We will find that if a nuclide can lower its energy by undergoing disintegration, it will do so – this is the simple explanation of radioactivity. Notice the proton is lighter than the neutron, suggesting the former is more stable than the latter. Indeed, if the neutron is not bound in

a nucleus (that is, it is a free neutron) it will decay into a proton plus an electron (plus an antineutrino) with a half-life of about 13 min.

Nuclear masses have been determined to quite high accuracy, precision of ~ 1 part in 10^8, by the methods of mass spectrography and energy measurements in nuclear reactions. Using the mass data alone we can get an idea of the stability of nuclides. Define the mass defect as the difference between the actual mass of a nuclide and its mass number, $\Delta = M - A$; it is also called the "mass decrement". If we plot Δ versus A, we get a curve sketched in Fig. 1. When $\Delta < 0$, it means that taking the individual nucleons when
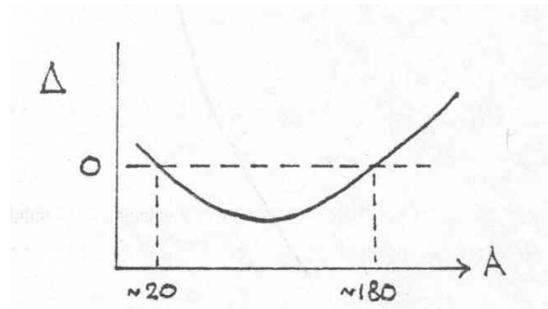

Fig. 1. Variation of mass decrement (M-A) showing that nuclides with mass numbers in the range ~ (20-180) should be stable.

they are separated far from each other to make the nucleus gives a product whose mass is lighter than the sum of the components. This can only happen if energy is given off during the formation. In other words, to reach a final state (the product nuclide) with smaller mass than the initial state (collection of individual nucleons) one must take away some energy (mass). It also follows that the final state will be more stable than the initial state, since energy must be put back in if one wants to reverse the process to go from the nuclide to the individual nucleons. We therefore expect that $\Delta < 0$ means the nuclide is stable. Conversely, $\Delta > 0$ means the nuclide is unstable. Our sketch shows that very light elements (A < 20) and heavy elements (A > 180) are not stable, and that maximum stability occurs around A ~ 50. We will return to discuss this behavior further when we consider the nuclear binding energy later.

Nuclear Size

According to Thomson's "electron" model of the nucleus (~ 1900), the size of a nucleus should be about an *Angstrom*, 10^{-8} cm. We now know this picture is wrong. The correct nuclear size was determined by Rutherford (~ 1911) in his atomic nucleus hypothesis which put the size at about 10^{-12} cm. Nuclear radius is not well defined, strictly speaking, because any result of a measurement depends on the phenomenon involved (different experiments give different results). On the other hand, all the results agree qualitatively and to some extent also quantitatively. Roughly speaking, we will take the nuclear radius to vary with the 1/3 power of the mass number, $R = r_o A^{1/3}$, with $r_o \sim 1.2 - 1.4 \times 10^{-13}$ cm. The lower value of the coefficient r_o comes from electron scattering which probes the charge distribution of the nucleus, while the higher value comes from nuclear scattering which probes the range of nuclear force. Since nuclear radii have magnitudes of the order of 10^{-13} cm, it is conventional to adopt a length unit called Fermi (F), $F \equiv 10^{-13}$ cm. Because of particle-wave duality we associate a wavelength with the momentum of a particle. The corresponding wave is called the *deBroglie* wave. Before discussing the connection between a wave property, the wavelength, and a particle property, the momentum, let us first set down the relativistic kinematic relations between mass, momentum and energy of a particle with arbitrary velocity. Consider a particle with rest mass m_o moving with velocity v. There are two expressions we can write down for the total energy E of this particle. One is the sum of its kinetic energy E_{kin} and its rest mass energy, $E_o = m_o c^2$,

$$E_{tot} = E_{kin} + E_o = m(v)c^2 \tag{1.1}$$

The second equality introduces the relativistic mass m(v) which depends on its velocity,

$$m(v) = \gamma m_o, \qquad \gamma = (1 - v^2/c^2)^{-1/2} \tag{1.2}$$

where γ is the Einstein factor. To understand (1.2) one should look into the Lorentz transformation and the special theory of relativity in any text. Eq.(1.1) is a first-order relation for the total energy. Another way to express the total energy is a second-order relation

$$E^2 = c^2 p^2 + E_o^2 \tag{1.3}$$

where p = m(v)v is the momentum of the particle. Eqs. (1.1) – (1.3) are the general relations between the total and kinetic energies, mass, and momentum. We now introduce the deBroglie wave by defining its wavelength λ in terms of the momentum of the corresponding particle,

$$\lambda = h / p \tag{1.4}$$

where h is the *Planck's* constant ($h / 2\pi = \hbar = 1.055 x 10^{-27}$ erg sec). Two limiting cases are worth noting.

Non-relativistic regime:

$$E_o \gg E_{kin}, \qquad p = (2 m_o E_{kin})^{1/2}, \qquad \lambda = h / \sqrt{2 m_o E_{kin}} = h / m_o v \tag{1.5}$$

Extreme relativsitic regime:

$$E_{kin} \gg E_o, \qquad p = E_{kin} / c, \qquad \lambda = hc / E \tag{1.6}$$

Eq.(1.6) applies as well to photons and neutrinos which have zero rest mass. The kinematical relations just discussed are general. In practice we can safely apply the non-relativistic expressions to neutrons, protons, and all nuclides, because their rest mass energies are always much greater than any kinetic energies we will encounter. The same is not true for electrons, since we will be interested in electrons with energies in the Mev region. Thus, the two extreme regimes do not apply to electrons, and one should use (1.3) for the energy-momentum relation. Since photons have zero rest mass, they are always in the relativistic regime.

Nuclear charge

The charge of a nuclide $_z X^A$ is positive and equal to Ze, where e is the magnitude of the electron charge, e = 4.80298 x 10^{-10} esu (= 1.602189 x 10^{-19} Coulomb). We consider single atoms as exactly neutral, the electron-proton charge difference is < 5 x 10^{-19} e, and the charge of a neutron is < 2 x 10^{-15} e.

We can look to high-energy electron scattering experiments to get an idea of how nuclear density and charge density are distributed across the nucleus. Fig. 2 shows two typical nucleon density distributions obtained by high-energy electron scattering. One
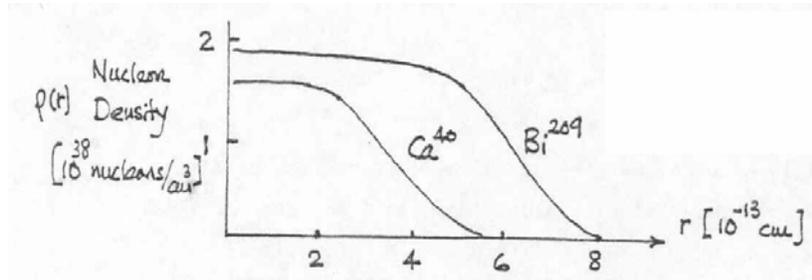

Fig. 2. Nucleon density distributions showing nuclei having no sharp boundary.

can see two basic components in each distribution, a core region of constant density and a boundary region where the density decreases smoothly to zero. Notice the magnitude of the nuclear density is 10^{38} nucleons per cm^3, compared to the atomic density of solids and liquids which is in the range of 10^{24} nuclei per cm^3. What does this say about the packing of nucleons in a nucleus, or the average distance between nucleons versus the separation between nuclei? Indeed the nucleons are packed together much more closely than the nuclei in a solid. The shape of the distributions shown in Fig. 2 can be fitted to the expression, called the Saxon distribution,

$$\rho(r) = \frac{\rho_o}{1 + \exp[(r - R)/a]} \tag{1.7}$$

where $\rho_o = 1.65 \times 10^{38}$ nucleons/cm^3, R ~ 1.07 A$^{1/3}$ F, and a ~ 0.55 F. A sketch of this distribution, given in Fig. 3, shows clearly the core and boundary components of the
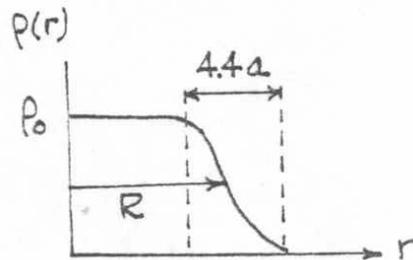

Fig. 3. Schematic of the nuclear density distribution, with R being a measure of the nuclear radius, and the width of the boundary region being given by 4.4a.

distribution. Detailed studies based on high-energy electron scattering have also revealed that even the proton and the neutron have rather complicated structures. This is illustrated in Fig. 4. We note that mesons are unstable particles of mass between
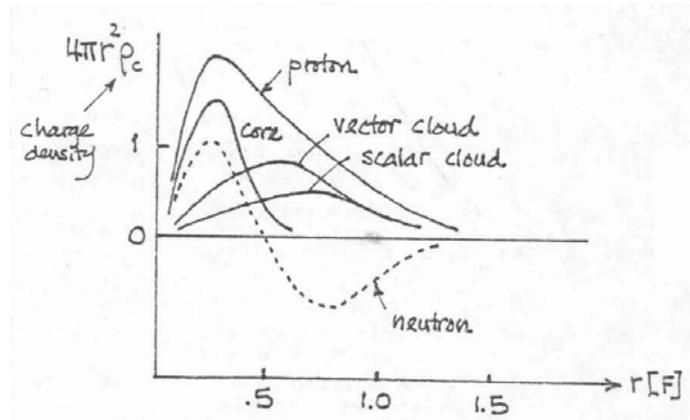

Fig. 4. Charge density distributions of the proton and the neutron showing how each can be decomposed into a core and two meson clouds, inner (vector) and outer (scalar). The core has a positive charge of ~0.35e with probable radius 0.2 F. The vector cloud has a radius 0.85 F, with charge .5e and -.5e for the proton and the neutron respectively, whereas the scalar clouid has radius 1.4 F and charge .15e for both proton and neutron[adopted from Marmier and Sheldon, p. 18].

the electron and the proton: π-mesons (pions) play an important role in nuclear forces ($m_\pi \sim 270 m_e$), μ-mesons(muons) are important in cosmic-ray processes ($m_\mu \sim 207 m_e$).

Nuclear Spin and Magnetic Moment

Nuclear angular momentum is often known as nuclear spin $\hbar \underline{I}$; it is made up of two parts, the intrinsic spin of each nucleon and their orbital angular momenta. We call I the spin of the nucleus, which can take on integral or half-integral values. The following is usually accepted as facts. Neutron and proton both have spin 1/2 (in unit of $\hbar$). Nuclei with even mass number A have integer or zero spin, while nuclei of odd A have half-integer spin. Angular momenta are quantized.

Associated with the spin is a magnetic moment $\underline{\mu}_I$, which can take on any value because it is not quantized. The unit of magnetic moment is the magneton

$$\mu_n \equiv \frac{|e|\hbar}{2m_p c} = \frac{\mu_B}{1836.09} = 0.505 \text{ x } 10^{-23} \text{ ergs/gauss} \qquad (1.8)$$

where μ_B is the Bohr magneton. The relation between the nuclear magnetic moment and the nuclear spin is

$$\underline{\mu}_I = \gamma \hbar \underline{I} \qquad (1.9)$$

where γ here is the gyromagnetic ratio (no relation to the Einstein factor in special relativity). Experimentally, spin and magnetic moment are measured by hyperfine structure (splitting of atomic lines due to interaction between atomic and nuclear magnetic moments), deflations in molecular beam under a magnetic field (Stern-Gerlach), and nuclear magnetic resonance (precession of nuclear spin in combined DC and microwave field). We will say more about nmr later.

Electric Quadruple Moment

The electric moments of a nucleus reflect the charge distribution (or shape) of the nucleus. This information is important for developing nuclear models. We consider a classical calculation of the energy due to electric quadruple moment. Suppose the nuclear charge has a cylindrical symmetry about an axis along the nuclear spin $\underline{I}$, see Fig. 5.
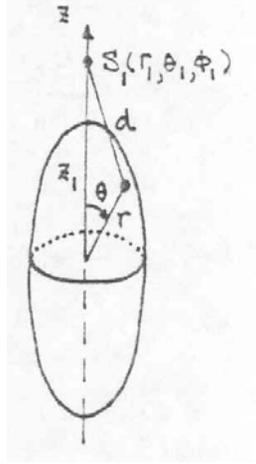
Fig. 5. Geometry for calculating the Coulomb potential energy at the field point S_1 due to a charge distribution $\rho(r)$ on the spheroidal surface as sketched. The sketch is for $\underline{r}_1$ located along the z – axis.

The Coulomb energy at the point S_1 is

$$V(r_1,\theta_1) = \int d^3r \frac{\rho(r)}{d} \tag{1.10}$$

where $\rho(r)$ is the charge density, and $d = |\underline{r}_1 - \underline{r}|$. We will expand this integral in a power series in $1/r_1$ by noting the expansion of $1/d$ in a Legendre polynomial series,

$$\frac{1}{d} = \frac{1}{r_1} \sum_{n=0}^{\infty} \left(\frac{r}{r_1}\right)^n P_n(\cos\theta) \tag{1.11}$$

where $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = (3x^2 - 1)/2$, …Then (1.10) can be written as

$$V(r_1,\theta_1) = \frac{1}{r_1} \sum_{n=0}^{\infty} \frac{a_n}{r_1^n} \tag{1.12}$$

with
$$a_o = \int d^3 r \rho(r) = Ze \qquad (1.13)$$

$$a_1 = \int d^3 r z \rho(r) = \text{electric dipole} \qquad (1.14)$$

$$a_2 = \int d^3 r \frac{1}{2}(3z^2 - r^2)\rho(r) \equiv \frac{1}{2} eQ \qquad (1.15)$$

The coefficients in the expansion for the energy, (1.12), are recognized to be the total charge, the dipole (here it is equal to zero), the quadruple, etc. In (1.15) Q is defined to be the quadruole moment (in unit of 10^{-24} cm^2, or barns). Notice that if the charge distribution were spherically symmetric, $<x^2> = <y^2> = <z^2> = <r^2>/3$, then Q = 0. We see also, Q > 0, if $3<z^2> > <r^2>$ and Q < 0, if $3<z^2> < <r^2>$. The corresponding shape of the nucleus in these two cases would be prolate or oblate spheroid, respectively (see Fig. 6).
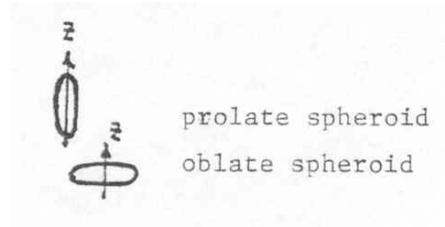

Fig. 6. Prolate and oblate spheroidal shapes of nuclei as indicated by a positive or negative value of the electric quadruple moment Q.

Some values of the spin and quadruple moments are:

Nucleus	I	Q [10^{-24} cm^2]
n	1/2	0
p	1/2	0
H^2	1	0.00274
He^4	0	0
Li^6	1	-0.002
U^{233}	5/2	3.4
U^{235}	7/2	4
Pu^{241}	5/2	4.9

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For calculations and quick estimates, the following table of physical constants and conversion factors, taken from Meyerhof, *Elements of Nuclear Physics* (1967), appendix D, is useful.

Constant	Value	Unit	
		mks	cgs
Speed of light in vacuum c	2.997925(1)	$\times 10^8$ m s^{-1}	$\times 10^{10}$ cm s^{-1}
Elementary charge e	1.60210(2) 4.80298(7)	10^{-19} C	10^{-20} emu 10^{-10} esu
Avogadro's number N	6.02252(9)	10^{26} kmole^{-1}	10^{23} mole^{-1}
Mass unit	1.66043(2)	10^{-27} kg	10^{-24} g
Electron rest mass m_0	9.10908(13) 5.48597(3)	10^{-31} kg 10^{-4} u	10^{-28} g 10^{-4} u
Proton rest mass M_P	1.67252(3) 1.00727663(8)	10^{-27} kg u	10^{-24} g u
Neutron rest mass M_n	1.67482(3) 1.0086654(4)	10^{-27} kg u	10^{-24} g u
Faraday constant Ne	9.64870(5) 2.89261(2)	10^4 C mole^{-1}	10^3 emu 10^{14} esu
Planck constant $\dfrac{h}{\hbar = h/2\pi}$	6.62559(16) 1.054494(25)	10^{-34} J s 10^{-34} J s	10^{-27} erg s 10^{-27} erg s
Charge-to-mass ratio for electron e/m_0	1.758796(6) 5.27274(2)	10^{11} C kg^{-1}	10^7 emu 10^{17} esu
Rydberg constant $2\pi^2 m_0 e^4 / h^3 c$	1.0973731(1)	10^7 m^{-1}	10^5 cm^{-1}
Bohr radius $\hbar^2/m_0 e^2$	5.29167(2)	10^{-11} m	10^{-9} cm
Compton wavelength of electron $\dfrac{h/m_0 c}{\hbar/m_0 c}$	2.42621(2) 3.86144(3)	10^{-12} m 10^{-13} m	10^{-10} cm 10^{-11} cm
Compton wavelength of proton $\dfrac{h/M_p c}{\hbar/M_p c}$	1.321398(13) 2.10307(2)	10^{-15} m 10^{-16} m	10^{-13} cm 10^{-14} cm

General Physical Constants. The numbers in parentheses after each quoted value represent the standard deviation error in the final digits of the given value, as computed on the criterion of internal consistency. The unified scale of atomic weights (12C = 12) is used throughout.

Figure by MIT OCW. Adapted from Meyerhof, Appendix D.

1 electron volt	1.6021×10^{-19} J
	1.6021×10^{-12} erg
1 erg	1 gm cm^2/sec
1 $\hbar$	1.054×10^{-27} erg sec
1k$_B$	1.3806×10^{-16} erg °K^{-1}

Proton mass	931.478	Mev
Neutron mass	939.550	Mev
Electron mass	0.511	Mev